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Closed Loop Subspace Identification

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Keywords: *Closed loop, subspace system identification, Kalman filter, unbiased estimates.*

A new three step closed loop subspace identifications algorithm based on an already existing algorithm and the Kalman filter properties is presented. The Kalman filter contains noise free states which implies that the states and innovation are uncorrelated. The idea is that a Kalman filter found by a good subspace identification algorithm will give an output which is sufficiently uncorrelated with the noise on the output of the actual process. Using feedback from the output of the estimated Kalman filter in the closed loop system a subspace identification algorithm can be used to estimate an unbiased model.

1. Introduction

Subspace identification (SID) algorithms assume the future inputs and the noise on the future outputs to be uncorrelated in order to produce unbiased estimates. If future inputs and the noise on the future outputs are correlated the projection of the future outputs onto the future inputs will cause a bias. Several methods and modifications to overcome this problem have been suggested, Van Overschee and De Moor (1996), (1997), Chou and Verhaegen (1997), Gustafsson (2001), Jansson (2003).

In this paper a new three step algorithm based on the DSR algorithm, Di Ruscio (1996), Di Ruscio (1997), and the Kalman filter properties, Jazwinski (1970), is presented. The rest of this paper is organized as follows. Some basic notations and matrix equations are presented inn Section 2. A new closed loop subspace identification algorithm is presented in Section 3. In Section 4 simulation examples are presented to illustrate the behavior of the algorithm. Some concluding remarks follows in Section 5.

2. Preliminary definitions

2.1. *Notation in the methods used*

In DSR there are four parameters g, n, L and J that can be chosen by the user. The structure parameter g is put to zero if there is no direct feedthrough from input to output, which have to be the case in closed loop identification. The identification horizon used to predict the number of states is specified by the parameter L. The model order is specified by the parameter n and is limited by the interval $1 \leq n \leq L \cdot m$, where m is the number of outputs. The number of time instants in the past horizon which is used defining the instrument variable matrix which are used to remove noise is specified by the parameter J.

In the Prediction Error Method (PEM) and N4SID implemented in the system identification toolbox in Matlab 6.5 $nk = 1$ is used if there is no direct feedthrough from input to output.

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2.2. *Notation and system definitions*

Consider the following linear discrete time invariant state space model given by:

$$x_{k+1} = Ax_k + Bu_k + v_k, \tag{1}$$

$$y_k = Dx_k + w_k, \tag{2}$$

where $x_k \in \mathbb{R}^n$ is the state vector, $u_k \in \mathbb{R}^r$ is the input vector, $y_k \in \mathbb{R}^m$ is the output vector, v_k is the process noise and w_k is the measurement noise at discrete time k. We choose to introduce $v_k = \tilde{C}\tilde{v}_k$ to make it easier to reconstruct the simulation examples presented in Section (4).

The model identified by the methods compared in this paper is the discrete time Kalman filter on innovation form:

$$\bar{x}_{k+1} = A\bar{x}_k + Bu_k + Ce_k, \tag{3}$$

$$y_k = D\bar{x}_k + Fe_k, \tag{4}$$

where $E(e_k e_k^T) = I$ and $\bar{x}_k \in \mathbb{R}^n$ is the predicted state vector. The Kalman filter is the minimum variance estimator. An alternate expression is:

$$\bar{x}_{k+1} = A\bar{x}_k + Bu_k + K\varepsilon_k, \tag{5}$$

$$y_k = \bar{y}_k + \varepsilon_k, \tag{6}$$

where $\varepsilon_k \in \mathbb{R}^m$ is the innovation, $\bar{y}_k = D\bar{x}_k$, $E(\varepsilon_k \varepsilon_k^T) = FF^T$ and $K = CF^{-1}$ is the Kalman filter gain.

As an alternative to using state space presentation the model can also be expressed by transfer functions. Introducing the forward shift operator, z, given by $zu_k = u_{k+1}$, we can present the model by:

$$y_k = H^d(z)u_k + H^s(z)\varepsilon_k, \tag{7}$$

where $H^d(z)$ is the transfer function from the input, u_k, to the output, y_k. $H^s(z)$ is the transfer function from the innovation, ε_k, to the output, y_k. In case of multiple input multiple output systems $H^d(z)$ and $H^s(z)$ are matrices of transfer functions.

Throughout the paper we will denote the eigenvalues of the system $\lambda(A)$, the eigenvalues of the Kalman filter $\lambda(A - KD)$, the deterministic transition zeros of the system $\rho(H^d(z))$, the deterministic steady state gain matrix $H^d(1)$ and the stochastic steady state gain matrix $H^s(1)$.

3. Closed Loop Subspace Identification Algorithm

Di Ruscio (2003) have presented an expression for the error term in the projection used to estimate the extended observability matrix in DSR. This term is approximately zero for open loop problems. The error term contains the projection of future output noise onto the future inputs. In some closed loop problems the error term is non-zero and cause biased estimates. This is the problem in feedback systems where the control is directly proportional to the innovations noise. It is stated by Di Ruscio (2003) that *It is believed that SID of the systems with state feedback or feedback from Kalman filter states would work well, provided an external dither signal is introduced in the loop. The reason for this is that the states are "noise-free" and not correlated with the innovations noise. There are no problems by using subspace identification methods in these cases.*

By definition the minimum variance estimator (Kalman filter) minimize the error norm $E((x_k - \bar{x}_k)(x_k - \bar{x}_k)^T)$. The Orthogonal Projection Lemma, Jazwinski (1970), gives a condition that is equivalent to the minimization of the error norm. The Orthogonal Projection Lemma states that the error is orthogonal to the approximation space Y_k. In the Kalman filter problem the approximation space is $Y_k = \{space\ spanned\ by\ y_1, \ldots, y_k\}$. This means that:

$$E(\bar{y}_k \varepsilon_k^T) = 0. \tag{8}$$

Therefore we want to suggest a new algorithm based on feedback from a Kalman filter found by DSR in closed loop. Algorithm for closed loop SID:

Algorithm 3.1

- *Step 1. Identification of Kalman filter using DSR*
- *Step 2. Implementation of the Kalman filter identified in Step 1*
- *Step 3. Identification of unbiased model using DSR*

The model found by DSR in Step 1 may have a bias, when the system is operating in closed loop and there is noise present. The output, $\bar{y}_k$, from a non-optimal Kalman filter will have some level of correlation to the innovation, ε_k. The idea is that the Kalman filter found by DSR in Step 1 will give an output, $\bar{y}_k$, which is sufficiently uncorrelated with the noise on the output of the actual process, and in this way reduce or eliminate the bias problem.

In this work a simulation study is performed. The simulation study consist of a single input single output system, Section 4.1, and a multiple input multiple output system, Section 4.2. The results from these simulation studies are commented. However, Step 1 in the algorithm may have a bias. Due to the bootstrap structure of the algorithm it may be advantageous to introduce a Step 4 to make the algorithm iterative. Prior to introducing iteration a convergence analysis have to be performed. This is a topic of a future work, and it will be natural to look for connections with convergence analysis of so called bootstrap instrumental variable methods, Söderström and Stoica (1983).

A block diagram of the algorithm is shown in Figure (1). In Step 1 the switch in the figure is in position 1. In all other cases the switch is in position 2. Section 4 contains examples where this method is used.

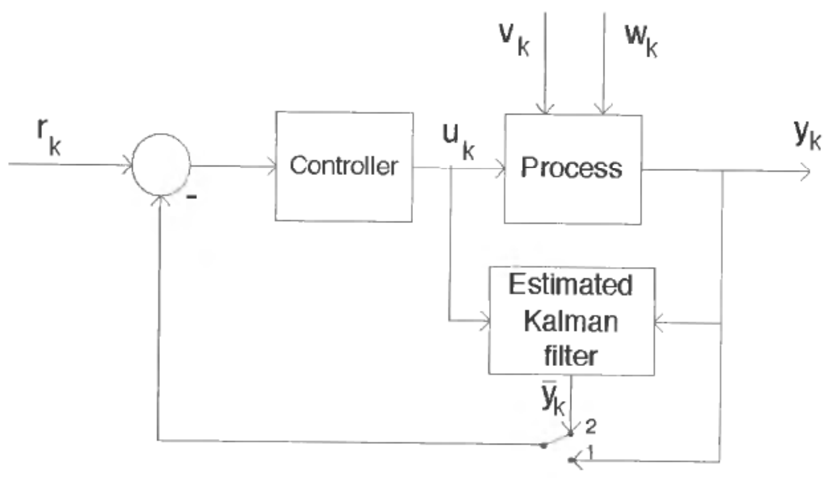

Figure 1. Block diagram of the algorithm. The switch is in position 1 in Step 1, else the switch is in position 2.

4. Simulation examples

4.1. *Single Input Single Output simulation example*

A single input single output system given by:

$$A=\begin{bmatrix} 0 & 1 \\ -0.7 & 1.5 \end{bmatrix},\ B=\begin{bmatrix} 0.25 \\ 0.625 \end{bmatrix},\ \tilde{C}=\begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix},$$

$$D=[1 \quad 0], \tag{9}$$

controlled by a PI-controller with $k_p=2$ and $T_i=5$ is used as an example. The PI-controller on discrete form is given by:

$$u_k=k_p\epsilon_k+z_k, \tag{10}$$

$$z_{k+1}=z_k+\frac{k_p}{T_i}\epsilon_k, \tag{11}$$

where ϵ_k is given by:

$$\epsilon_k=r_k-y_k. \tag{12}$$

The process noise variance used is $E(\tilde{v}_k\tilde{v}_k^T)=0.01$ and the measuring noise variance used is $E(w_k^2)=0.01$. Time series of N = 1000 discrete data points, $k=0,1,\ldots,N$, are generated.

A (Pseudo Random Binary Signal) PRBS of length N is used as reference to excite the process. To illustrate the noise level and the PRBS used as reference, the reference, r_k^1, is plotted in Figure 2 with the corresponding input, u_k, and the output, y_k, for two particular noise realizations.

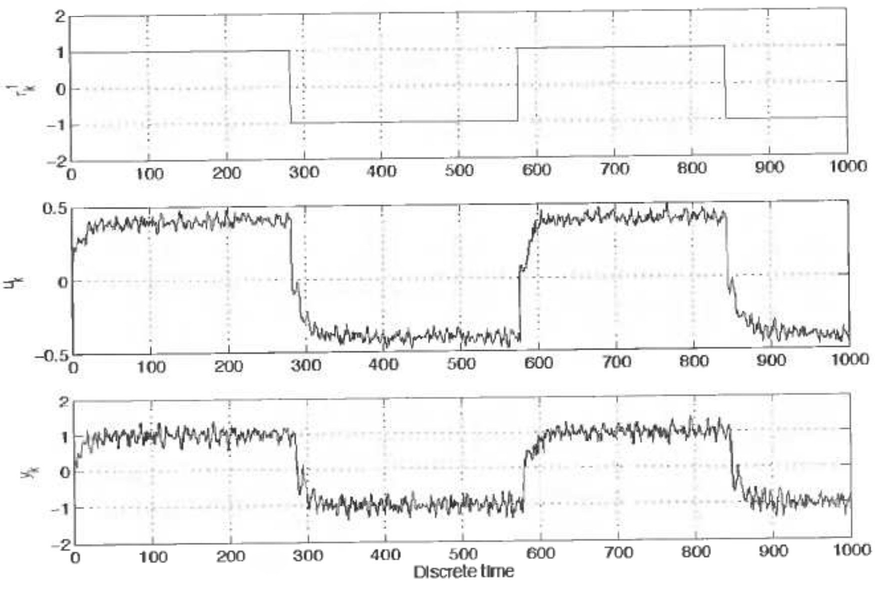

Figure 2. The reference signal, r_k^1, with corresponding input, u_k, and output, y_k, for two particular noise realizations, v_k and w_k.

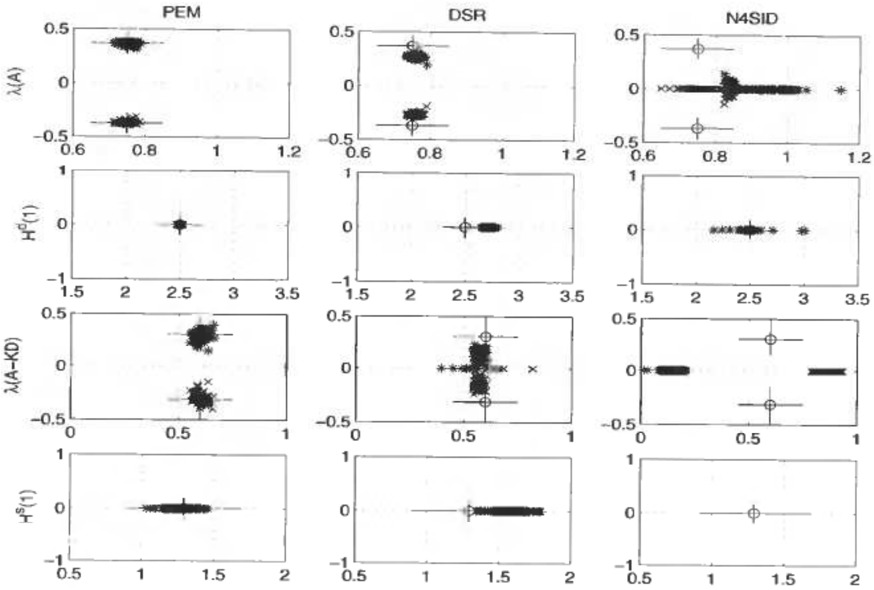

Figure 3. Estimates from closed loop Monte Carlo simulation using r_k^1 as reference.

To visualize the bias problem that occurs when SID algorithms is used for direct closed loop identification a Monte Carlo simulation with 100 runs using r_k^1 as reference signal with different noise realization in each run is carried out. The system order $n = 2$ and the fact that there is no direct feedthrough term from input to output is assumed known. The SID algorithms DSR and N4SID are compared to PEM. The N4SID version and the PEM version used is the implementations in Matlab 6.5. PEM and N4SID are used with default parameters and $nk = 1$. DSR is used with $g = 0$, $L = 5$ and $J = 6$. Figure 3 shows the estimates.

The estimates from PEM are unbiased and the estimates from the SID algorithms are biased. Comparing the SID algorithms it is clear that the estimates from N4SID have a much larger bias than the estimates from DSR. Therefore it is not advisable to use N4SID in Step 1 in the algorithm introduced in Section 3.

To evaluate the quality of the algorithm introduced in Section 3, Step 1 is performed by a single simulation using r_k^1 as reference to identify a (biased) model using DSR. The Kalman filter found by DSR with $g = 0$, $L = 5$ and $J = 6$ is given by:

$$A = \begin{bmatrix} 0 & 1 \\ -0.6508 & 1.5257 \end{bmatrix}, \quad B = \begin{bmatrix} 0.9134 \\ 0.8175 \end{bmatrix},$$

$$D = [1 \quad 0], \quad K = [0.3634 \quad 0.2675]^T. \tag{13}$$

To illustrate the effect on the noise level when using feedback filtered through the Kalman filter found by DSR, Equation (13), the reference r_k^1 is plotted in Figure 4 with the corresponding input, u_k, and the output, y_k, for two particular noise realizations with the same noise level as in the previous simulations.

The noise level is of course reduced since the input is a function of the filtered output, but this was not the main goal. The main goal was to generate an input, u_k, which is uncorrelated with the noise on the output, y_k.

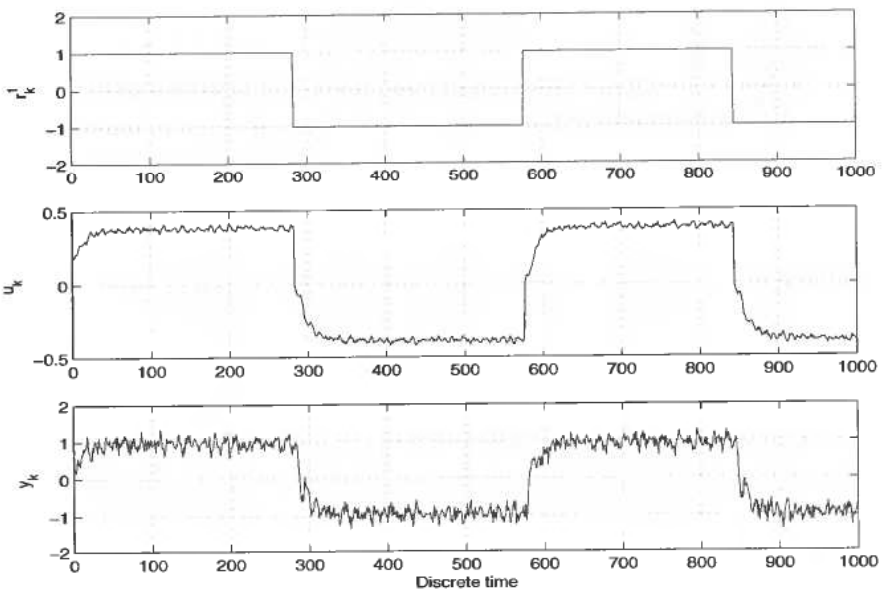

Figure 4. The reference signal, r_k^1, with corresponding input, u_k, and output, y_k, for two particular noise realizations, v_k and w_k, when the feedback is filtered through a Kalman filter found by DSR, Equation (13), in an initial step.

Step 2 and 3 of the algorithm introduced in Section 3 is evaluated by a Monte Carlo simulation with 100 runs with different noise realization in each run carried out using r_k^1 as reference signal, when the feedback is filtered through the Kalman filter found by DSR, Equation (13), in an initial simulation. As in previous simulations the system order $n = 2$ is assumed known. PEM and N4SID are used with default parameters and $nk = 1$. DSR is used with $g = 0$, $L = 5$ and $J = 6$. Figure 5 shows the estimates with no iterations performed to improve the Kalman filter.

Now when the feedback is filtered through the Kalman filter found by DSR, Equation (13), all the methods give unbiased estimates, but the estimates from N4SID have considerable larger variance than the others. It is quite satisfactory that the estimates from PEM does not have any observable increase in variance when the feedback is filtered through the Kalman filter found by DSR, Figure (5), compared to direct closed loop identification, Figure (3). It indicates that the Kalman filter estimated in Step 1 in the algorithm does not have to be very accurate to have the desired effect.

We are pleased to observe that the control function still is satisfactory when the feedback is filtered through the Kalman filter found by DSR in the initial step.

Figure (6) shows the reference r_k^1 with the corresponding input, u_k, and the output, y_k, for two particular noise realizations with the same noise level as in the previous simulations when the feedback is filtered through the correct Kalman filter.

Figure (7) shows the estimates when the feedback is filtered through the correct Kalman filter. It have to be noted that there is no significant improvement of the performance compared to when the feedback is filtered through the Kalman filter found by DSR in an initial step, Figure (5).

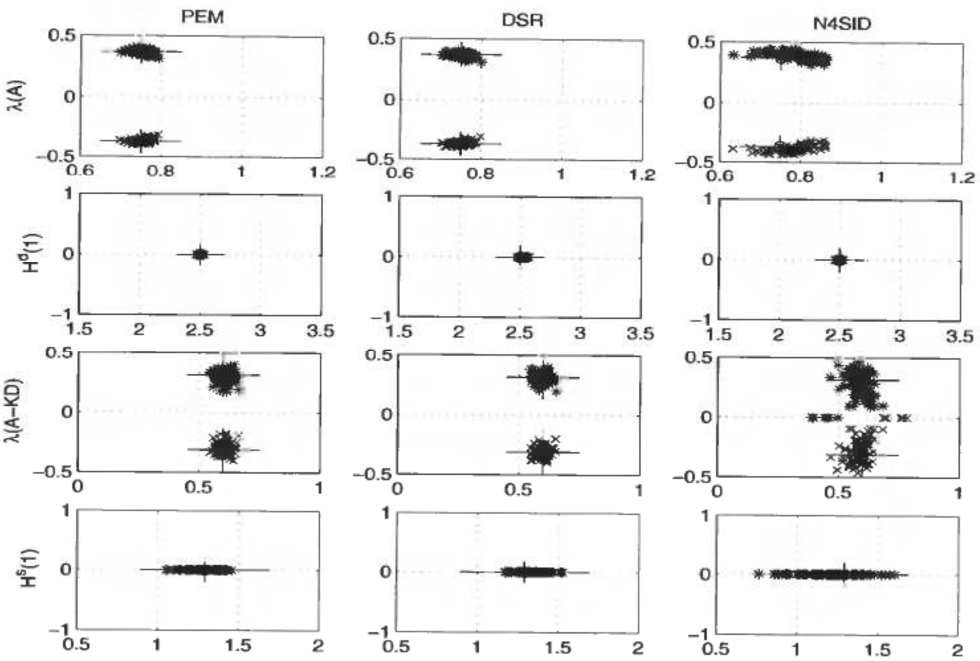

Figure 5. Estimates from closed loop Monte Carlo simulation using r_k^1 as reference when the feedback is filtered through a Kalman filter found by DSR, Equation (13), in an initial step.

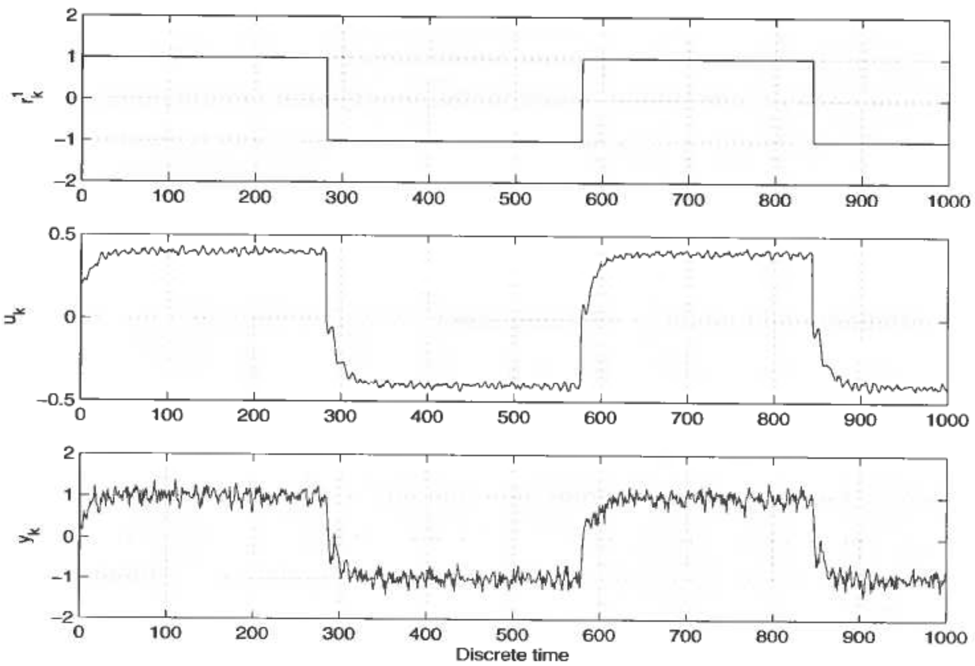

Figure 6. The reference signal, r_k^1, with corresponding input, u_k, and output, y_k, for two particular noise realizations, v_k and w_k, when the feedback is filtered through the correct Kalman filter.

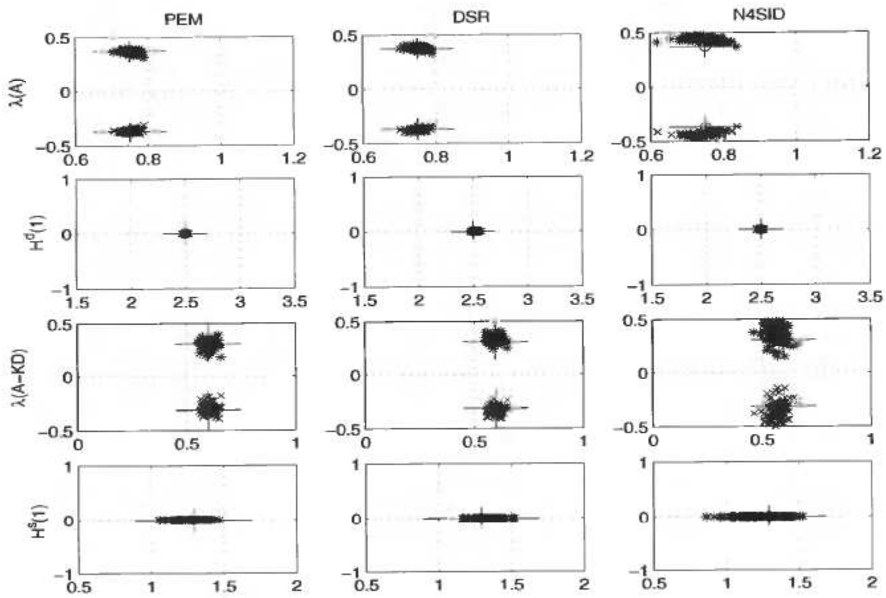

Figure 7. Estimates from closed loop Monte Carlo simulation using r_k^1 as reference when the feedback is filtered through the correct Kalman filter.

4.2. *Multiple Input Multiple Output simulation example*

A multiple input multiple output system is given by:

$$A=\begin{bmatrix}1.5 & 1.0 & 0.1\\ -0.7 & 0.0 & 0.1\\ 0 & 0 & 0.85\end{bmatrix},\ B=\begin{bmatrix}0 & 0\\ 0 & 1\\ 1 & 0\end{bmatrix},\ \tilde{C}=\begin{bmatrix}0.0 & 0.1\\ 0.1 & 0\\ 0 & 0.2\end{bmatrix},$$

$$D=\begin{bmatrix}3 & 0 & -0.6\\ 0 & 1 & 1\end{bmatrix},\ E=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}. \tag{14}$$

Input 1 is used to control output 1 using a PI-controller with $Kp=0.02$ and $Ti=2$. Input 2 is used to control output 2 using a PI-controller with $Kp=-0.02$ and $Ti=2$. The PI-controllers used to control the outputs are given by (10) and (11). The process noise variance used is

$$E(\tilde{v}_k\tilde{v}_k^T)=\begin{bmatrix}0.001 & 0\\ 0 & 0.001\end{bmatrix}$$

and the measuring noise variance used is

$$E(w_k w_k^T)=\begin{bmatrix}0.001 & 0\\ 0 & 0.0005\end{bmatrix}.$$

Time series of N = 1000 discrete data points, $k=0,1,\ldots,N$, are generated. As reference signal for output 1 and output 2 respectively r_k^2 and r_k^3 are used. Figure 8 shows the reference signals, r_k^2 and r_k^3, plotted together with the corresponding outputs for two particular noise realizations, v_k and w_k, when the system is operating in closed loop.

In this example we assume known the system order $n=3$ and the fact that there is no direct feedthrough term from input to output. In figure (9) the estimates from a Monte Carlo simulation with 100 runs using r_k^2 and r_k^3 as reference signals with different noise realizations in each run where direct closed loop identification is

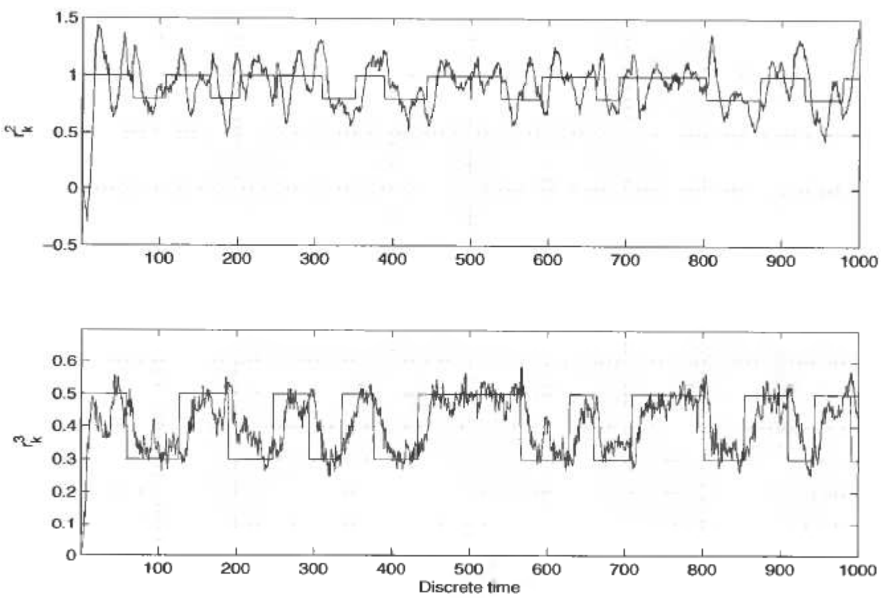

Figure 8. The reference signals, r_k^2 and r_k^3, plotted together with the corresponding outputs for two particular noise realizations, v_k and w_k.

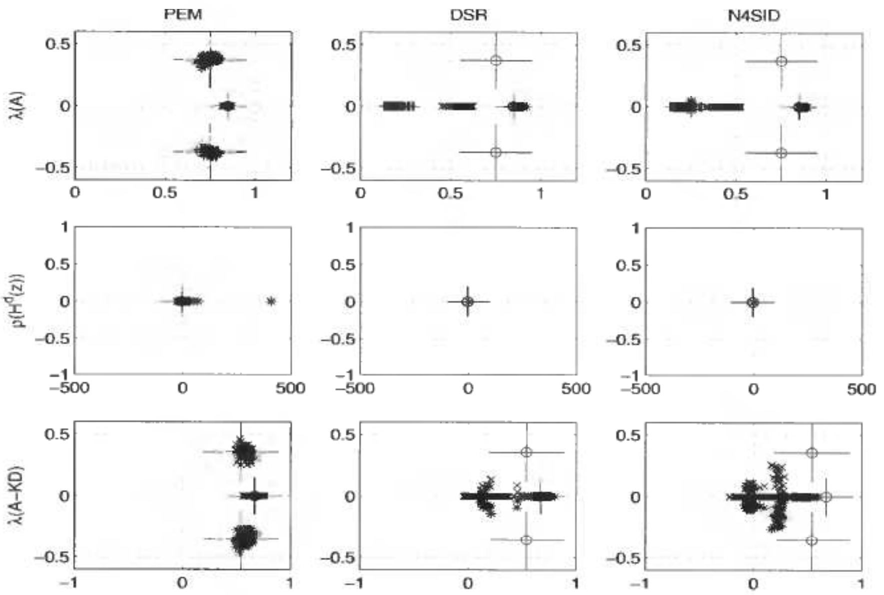

Figure 9. Estimates from closed loop Monte Carlo simulation using r_k^2 and r_k^3 as references.

performed. As in the single input single output case we observe that PEM give unbiased estimates and both the SID algorithms give biased estimates. Regarding the estimation of the zeros two comments have to be made. The first is that when the eigenvalue estimates from the SID algorithms are so poor, as they are here, there is just a coincidence that the estimation of the zeros, compared to PEM, is so good. The second is that when PEM have one or more estimates which seems like "outliers" the zeros are hard to estimate.

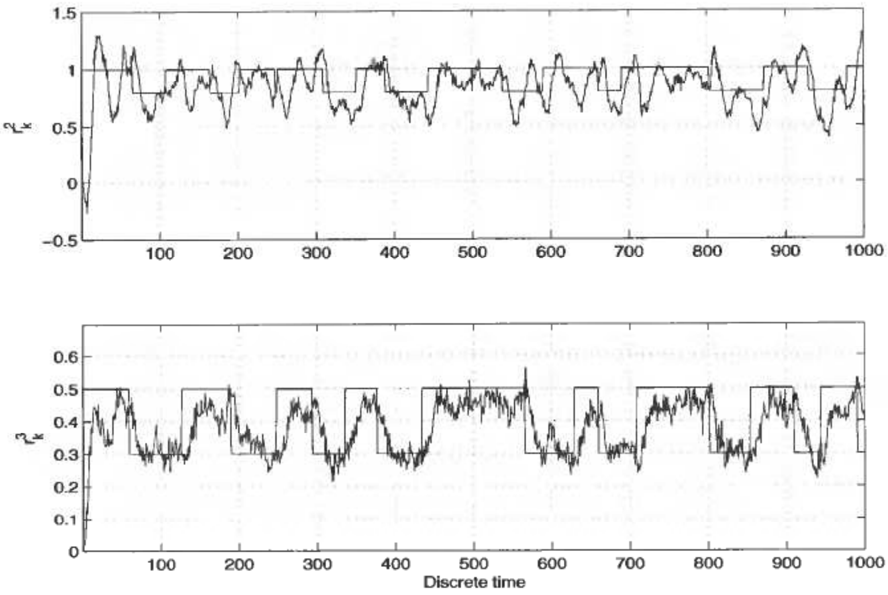

Figure 10. The reference signals, r_k^2 and r_k^3, with corresponding outputs, for two particular noise realizations, v_k and w_k, when the feedback is filtered through a Kalman filter found by DSR, Equation (15), in an initial step.

To evaluate the quality of the algorithm introduced in Section 3, Step 1 is performed by a single simulation using r_k^2 and r_k^3 as references to identify a (biased) model using DSR. The Kalman filter found by DSR with $g=0$, $L=8$ and $J=9$ is given by:

$$A=\begin{bmatrix} 0.7872 & 0.4514 & 0.2434 \\ 0.0435 & 0.6936 & -0.0203 \\ 0.0184 & 0.1157 & 0.1970 \end{bmatrix}, \qquad B=\begin{bmatrix} -4.1692 & -8.9423 \\ -2.2756 & 2.6672 \\ 1.7208 & -0.5849 \end{bmatrix},$$

$$D=\begin{bmatrix} -0.6301 & 0.5400 & 0.3128 \\ 0.0072 & -0.3117 & -0.8906 \end{bmatrix}, \; K=\begin{bmatrix} -0.7383 & -0.3019 \\ -0.0802 & -0.1532 \\ -0.0350 & 0.2059 \end{bmatrix}. \qquad (15)$$

To illustrate the effect on the noise level when using feedback filtered through the Kalman filter found by DSR, Equation (15), the references r_k^2 and r_k^3 are plotted in Figure 10 with the corresponding outputs, for two particular noise realizations with the same noise level as in the previous simulations. There is no significant reduction in the noise level. This is not a problem because the goal is to generate a feedback which is sufficiently uncorrelated with the noise on the output of the actual process.

Now when the feedback is filtered through the Kalman filter found by DSR, Equation (15), all the methods give unbiased estimates, but the estimates from N4SID have considerable larger variance than the others. It is quite satisfactory that the estimates from PEM does not have any observable increase in variance, except of the zeros, when the feedback is filtered through the Kalman filter found by DSR, Figure (10), compared to direct closed loop identification, Figure (9). It supports the observations in the single input single output example, Section (4.1), that the Kalman filter estimated in Step 1 in the algorithm does not have to be very accurate to have the desired effect.

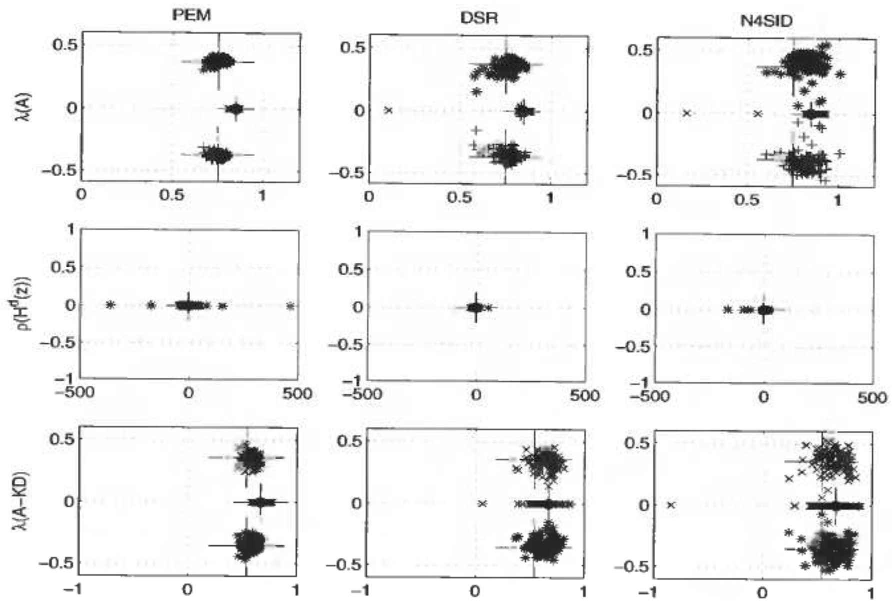

Figure 11. Estimates from closed loop Monte Carlo simulation using r_k^2 and r_k^3 as references when the feedback is filtered through a Kalman filter found by DSR, Equation (15), in an initial step.

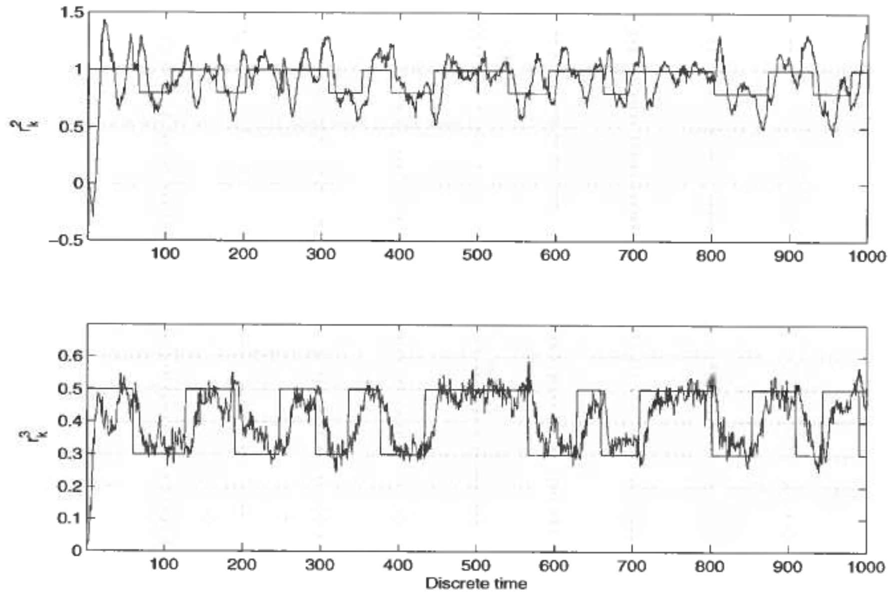

Figure 12. The reference signals, r_k^2 and r_k^3, with corresponding outputs, for two particular noise realizations, v_k and w_k, when the feedback is filtered through the correct Kalman filter.

Like in the example in Section (4.1) we observe that the control function still is satisfactory when the feedback is filtered through the Kalman filter found by DSR in the initial step.

Figure (12) shows the references r_k^2 and r_k^3 with the corresponding outputs for two particular noise realizations with the same noise level as in the previous simulations when the feedback is filtered through the correct Kalman filter.

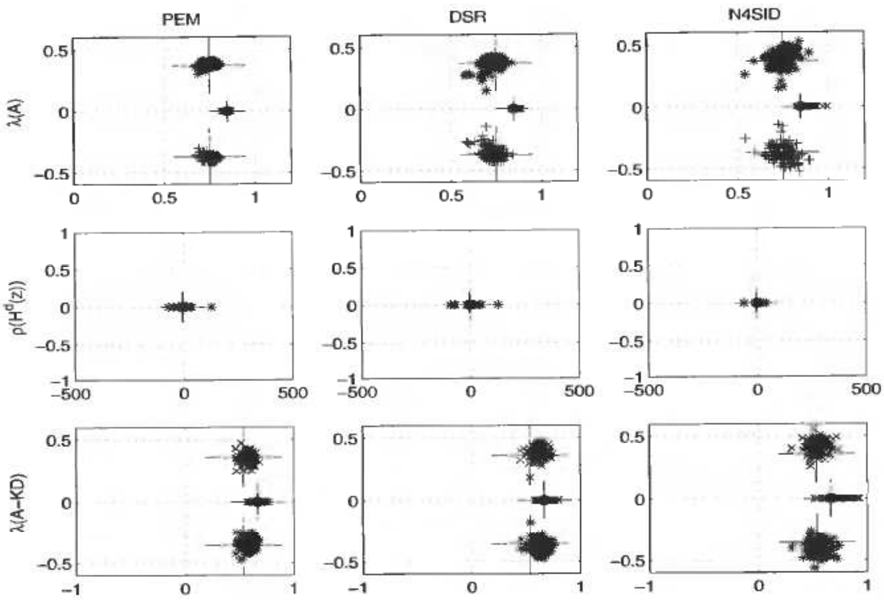

Figure 13. Estimates from closed loop Monte Carlo simulation using r_k^2 and r_k^3 as references when the feedback is filtered through the correct Kalman filter.

Figure (13) shows the estimates when the feedback is filtered through the correct Kalman filter. Like in Section (4.1) it have to be noted that there is no significant improvement of the performance compared to when the feedback is filtered through the Kalman filter found by DSR in an initial step, Figure (11).

5. Concluding remarks

A new three step closed loop subspace identifications algorithm based on the DSR algorithm and the Kalman filter properties is presented. In an initial step DSR is used for identification of the process model, including the Kalman filter gain. This model may have a bias when the system is operating in closed loop and there is noise present. The next step is to implement the Kalman filter in the feedback in such a way that the controller uses the filtered output from the filter, not the actual process measurement. The idea is that the Kalman filter found by DSR will give an output which is sufficiently uncorrelated with the noise on the output of the actual process, and in this way reduce or eliminate the bias problem. The final step is to use DSR to identify the process model when the feedback is filtered through the Kalman filter. This model will be unbiased if the Kalman filter is correct.

Our simulation studies have shown that even when a Kalman filter with a bias is used the estimated model in the final step is unbiased. A convergence analysis of the algorithm will be a topic of future work.

The initial idea was that any subspace identification algorithm which estimates the full state space model, inclusive the Kalman filter gain, should be applicable for this algorithm. The simulation study performed showed that it is not advisable to use N4SID in the initial step in the algorithm due to poor results. N4SID can be used in the final step in the algorithm, but it is not advisable because the variance is much larger than when DSR is used.

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