MODELING, IDENTIFICATION AND CONTROL, 1994, VOL. 15, NO. 4, 253–262
doi:10.4173/mic.1994.4.4

Generalized predictive control of nonlinear systems of the Hammerstein form

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Keywords: *Nonlinear systems, Hammerstein models, predictive control, adaptive control, stability analysis.*

A nonlinear generalized predictive control algorithm based upon a Hammerstein model is presented. Stability of the closed-loop system is analyzed with a control horizon equal to one. An adaptive nonlinear generalized predictive control algorithm with a linear estimator is then proposed. Finally, some results from simulation experiments are presented in order to show the algorithm's ability.

1. Introduction

Generalized predictive control (GPC) based upon linear models has enjoyed growing attention in the last few years, see, e.g., Clarke *et al.* (1987), Clark and Mohtadi (1989), De Keyser and Van Cauwenberghe (1985), De Keyser *et al.* (1988), Krämer and Unbehauen (1988), Lelic and Zarrop (1987), Wang and Henriksen (1992a, 1992b), and Ydstie (1985). Experimental studies and practical applications have demonstrated that satisfactory control performance can be obtained using GPC. However, most plants and systems to be controlled have some kind of nonlinearity, so there is definitely need to extend GPC design methods to nonlinear systems.

One extension of that kind appears in Zhu *et al.* (1991) where a GPC algorithm was used to control a plant described by a Hammerstein model. Due to the fact that the linear and non-linear parts of the system were considered separately in the latter, the stability of the closed-loop system was hard to analyze. Besides, a nonlinear estimation scheme had to be used in their algorithm. In this paper we will derive a nonlinear GPC algorithm based upon a Hammerstein model of the underlying system. Somewhat different from the work presented in Zhu *et al.* (1991) we will use a new cost function for the controller design. Stability analysis of the closed-loop system will be carried out with the control horizon equal to one. An adaptive nonlinear GPC algorithm with a linear estimation scheme will then be proposed, and some results from simulation experiments are presented at the end.

2. Controller design

The plant which is about to be controlled is assumed to be representable by a Hammerstein model of the form

$$Ay_t = Bx_{t-1} + C\omega_t/\Delta \tag{1}$$

Received 6 May 1994.
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This paper was presented at the European Control Conference, Groningen, June 28–1 July, 1993.

where A, B, C and Δ are polynomials in the backward shift operator z^{-1} of the form

$$\begin{aligned} A &= 1 + a_1 z^{-1} + \ldots + a_n z^{-n} \\ B &= b_0 + b_1 z^{-1} + \ldots + b_m z^{-m} \\ C &= 1 + c_1 z^{-1} + \ldots + c_l z^{-l} \quad \Delta = 1 - z^{-1} \end{aligned} \tag{2}$$

whereas the static nonlinearity is given by

$$x_t = r_0 + r_1 u_t + r_2 u_t^2 + \ldots + r_p u_t^p \tag{3}$$

where p is an odd number. $\{u_t\}$ and $\{y_t\}$ are the input and output processes, respectively. The model form depicted in (1) has the advantage that the controller derived in what follows will contain an integrator. $\{\omega_t\}$ is a stochastic process defined on a probability space $(\Omega, \mathscr{F}, \mathscr{P})$ on which we have a sequence $(\mathscr{F}_t, t \in N)$ of increasing σ-algebras where $\mathscr{F}_t$ is generated by the observations up to and including t. The process $\{\omega_t\}$ is assumed to satisfy, with probability 1,

$$\begin{aligned} &E\{\omega_t | \mathscr{F}_{t-1}\} = 0 \qquad E\{\omega_t^2 | \mathscr{F}_{t-1}\} = \sigma^2 \\ &\lim_{n \to \infty} \sup \left(\frac{1}{N} \sum_{t=1}^{N} \omega_t^2 \right) < \infty \end{aligned} \tag{4}$$

The cost function has the following form

$$J = E\left\{ \sum_{j=1}^{N_1} [(y_{t+j} - y_{t+j}^r)^2 + \lambda(\Delta u_{t+j-1}^p)^2] | \mathscr{F}_t \right\} \tag{5}$$

where $\{y_t^r\}$ is a (known) bounded set-point sequence, N_1 is the prediction horizon whereas λ is a weighting constant. The conditional expectation in (5) is, as indicated, taken given data up to and including time t. The cost on Δu_t^p is physically meaningful because Δu_t^p is monotically decreasing. The above therefore allows us to penalize changes in the control action.

We will, for the sake of simplicity, now assume $C = 1$. It should be noted, however, that our method can readily cope with coloured noise. Now, using the following two polynomial identities

$$1 = F_j A \Delta + z^{-j} G_j \qquad B F_j = E_j + z^{-j} H_j \tag{6}$$

for $j = 1, 2, \ldots, N_1$ and where

$$\begin{aligned} F_j &= 1 + f_1 z^{-1} + \ldots + f_{j-1} z^{-j+1} \\ G_j &= g_0^j + g_1^j z^{-1} + \ldots + g_n^j z^{-n} \\ E_j &= e_0 + e_1 z^{-1} + \ldots + e_{j-1} z^{-j+1} \\ H_j &= h_0^j + h_1^j z^{-1} + \ldots + h_{m-1}^j z^{-m+1} \end{aligned} \tag{7}$$

we can write the plant equation (1) in the form

$$y_{t+j} = E_j \Delta x_{t+j-1} + G_j y_t + H_j \Delta x_{t-1} + F_j \omega_{t+j} \tag{8}$$

for $j = 1, 2, \ldots, N_1$.

Using (3) we can write the N_1 equations in (8) in vector form, viz.

$$\mathbf{y} = \mathbf{E} \sum_{i=1}^{p} r_i \mathbf{u}_i + \mathbf{G} y_t + \mathbf{H} \sum_{i=1}^{p} r_i \Delta u_{t-1}^i + \mathbf{F} \tag{9}$$

where

$$\mathbf{y}=[y_{t+1}\,y_{t+2}\ldots y_{t+N_1}]^{\mathrm{T}}$$

$$\mathbf{u}_i=[\Delta u_t^i\,\Delta u_{t+1}^i\ldots\Delta u_{t+N_1-1}^i]^{\mathrm{T}}$$

$$\mathbf{G}=[G_1\ldots G_{N_1}]^{\mathrm{T}}\qquad \mathbf{H}=[H_1\ldots H_{N_1}]^{\mathrm{T}}$$

$$\mathbf{F}=[F_1\omega_{t+1}\ldots F_{N_1}\omega_{t+N_1}]^{\mathrm{T}} \tag{10}$$

$$\mathbf{E}=\begin{bmatrix} e_0 & & & & \\ e_1 & e_0 & & & \\ e_2 & e_1 & e_0 & & \\ \cdot & \cdot & \cdot & & \\ e_{N_1-1} & e_{N_1-2} & \cdot & \cdot & e_0 \end{bmatrix}$$

Define

$$\mathbf{y}_r=[y_{t+1}^r\,y_{t+2}^r\ldots y_{t+N_1}^r]^{\mathrm{T}} \tag{11}$$

From the definitions above we can write (5) as

$$J=E\{(\mathbf{y}-\mathbf{y}_r)^{\mathrm{T}}(\mathbf{y}-\mathbf{y}_r)+\lambda\mathbf{u}_p^{\mathrm{T}}\mathbf{u}_p|\mathscr{F}_t\} \tag{12}$$

Substituting (9) into (12), differentiating J wrt. $\mathbf{u}_p$, and putting the result equal to zero yield

$$\left(r_p\mathbf{I}+\sum_{i=1}^{p-1}r_i\frac{\partial\mathbf{u}_i^{\mathrm{T}}}{\partial\mathbf{u}_p}\right)\mathbf{E}^{\mathrm{T}}\left(\mathbf{E}\sum_{i=1}^{p}r_i\mathbf{u}_i+\mathbf{G}y_t+\mathbf{H}\sum_{i=1}^{p}r_i\Delta u_{t-1}^i-\mathbf{y}_r\right)+\lambda\mathbf{u}_p=0 \tag{13}$$

Neglecting the dependence of $u_i\,(i=1,\ldots,p-1)$ on $\mathbf{u}_p$ we obtain

$$r_p\mathbf{E}^{\mathrm{T}}\left(\mathbf{E}\sum_{i=1}^{p}r_i\mathbf{u}_i+\mathbf{G}y_t+\mathbf{H}\sum_{i=1}^{p}r_i\Delta\mathbf{u}_{t-1}^i-\mathbf{y}_r\right)+\lambda\mathbf{u}_p=0 \tag{14}$$

which can be written as

$$r_p\mathbf{E}^{\mathrm{T}}\mathbf{E}\sum_{i=1}^{p-1}r_i\mathbf{u}_i+(r_p^2\mathbf{E}^{\mathrm{T}}\mathbf{E}+\lambda\mathbf{I})\mathbf{u}_p=r_p\mathbf{E}^{\mathrm{T}}\left(\mathbf{y}_r-\mathbf{G}y_t-\mathbf{H}\sum_{i=1}^{p}r_i\Delta u_{t-1}^i\right) \tag{15}$$

(15) constitutes a total of N_1 equations with N_1 unknowns. It is not, however, easy to find a solution for u_t. In the above, note that the control horizon and the output prediction horizon have been selected to to be the same, i.e., N_1. This is not a necessary requirement. A control horizon $N_u<N_1$ can be used, see Clarke *et al.* (1987). Furthermore, it is possible to obtain satisfactory control of most plants by putting the control horizon $N_u=1$, see again Clarke *et al.* (1987). We therefore adopt $N_u=1$ in our design of the controller, i.e., $\Delta u_{t+j}^i=0$ for $j=1,\ldots,N_1-1$. In this case $\mathbf{u}_i$ and $\mathbf{E}$ as defined by (10) will take the forms, respectively,

$$\mathbf{u}_i=\Delta u_t^i\quad \mathbf{E}=[e_0\,e_1\ldots e_{N_1-1}]^{\mathrm{T}} \tag{16}$$

From (16) and the fact that $z^{-1}u_t^i=u_{t-1}^i$ for $i=1,\ldots,$ we can rewrite (15) as

$$ke\sum_{i=1}^{p-1}r_iu_t^i+u_t^p=Py_{t+N_1}^r-\alpha y_t-\beta\sum_{i=1}^{p}r_i\Delta u_{t-1}^i+ke\sum_{i=1}^{p-1}r_iu_{t-1}^i+u_{t-1}^p \tag{17}$$

where

$$k=\frac{r_p}{r_p^2e+\lambda}\;e=\sum_{i=0}^{N_1-1}e_i^2 \tag{18}$$

whereas P, α, and β are polynomials in the backward shift operator z^{-1} given by (G_j and H_j are polynomials, see (7))

$$P=k(e_{N_1-1}+e_{N_1-2}z^{-1}+\ldots+e_0z^{-N_1+1})$$

$$\alpha=k\sum_{j=1}^{N_1}e_{j-1}G_j \quad \beta=k\sum_{j=1}^{N_1}e_{j-1}H_j \tag{19}$$

Equation (17) is a p'th order Hammerstein polynomial in u_t which is fairly easy to solve numerically in order to find u_t. For example, the improved root solving method given in Zhu *et al.* (1991) can be used. Note that a real root of minimum magnitude can always be found because p is odd.

3. Stability analysis

Let us rewrite (17) in the form

$$(ke+z^{-1}\beta)\sum_{i-1}^{p-1}r_i\Delta u_t^i+(1+z^{-1}r_p\beta)\Delta u_t^p=Py_{t+N_1}^r-\alpha y_t \tag{20}$$

From (1) and (3) we have

$$A\Delta y_t=z^{-1}B\sum_{i=1}^{p-1}r_i\Delta u_t^i+z^{-1}r_pB\Delta u_t^p+\omega_t \tag{21}$$

Lemma

Let a system be described by (T is a polynomial in z^{-1})

$$T\Delta u_t^p=Ay_{t+d}+B\sum_{j=1}^{p-1}r_j\Delta u_t^j+C\omega_t \tag{22}$$

and let ω_t satisfy (4). If T is stable, then for some positive integer d

$$\frac{1}{N}\sum_{t=1}^{N}(\Delta u_t^p)^2\leqslant\frac{K_1}{N}\sum_{t=1}^{N}y_{t+d}^2+K_2 \tag{23}$$

where $0<K_1<\infty$ and $0<K_2<\infty$.

Proof. See the Appendix.

Theorem

If the control law given by (17) is used with N_1 and λ chosen such that the polynomial

$$T=A\Delta(1+z^{-1}r_p\beta)+z^{-1}r_p\alpha B \tag{24}$$

is stable, then with probability 1

(1) The resulting closed-loop system will be stable in the sense that $\{\Delta u_t^i\}$, $i=1,\ldots,p$, and $\{y_t\}$ are sample mean square bounded.
(2) The control law (17) minimizes the cost function

$$J'=E\{[P(y_{t+N_1}-y_{t+N_1}^r)+\lambda'\Delta u_t^p]^2|\mathscr{F}_t\} \tag{25}$$

where $\lambda'=\lambda k/r_p$. Moreover, the minimum possible value of the quadratic cost function (25) is

$$\gamma^2=k^2\sigma^2\sum_{j=1}^{N_1}\left(\sum_{i=0}^{N_1-j}f_ie_{i+j-1}\right)^2 \tag{26}$$

(3) For constant reference y_t^r and with $\omega_t = 0$ it follows that

$$\lim_{t\to\infty}(y_t - y_t^r) = 0 \tag{27}$$

Proof. (1) Multiplying (20) by $A\Delta$ and $z^{-1}r_pB$ respectively and using (21) we obtain

$$T\Delta u_t^p = A\Delta P y_{t+N_1}^r - [A\Delta(ke + z^{-1}\beta) + z^{-1}\alpha B]\sum_{i=1}^{p-1} r_i \Delta u_t^i - \alpha\omega_t \tag{28}$$

$$Ty_t = z^{-1}r_pBPy_{t+N_1}^r + z^{-1}\frac{\lambda}{r_p}kB\sum_{i=1}^{p-1} r_i\Delta u_t^i + (1 + z^{-1}r_p\beta)\omega_t \tag{29}$$

If T is stable, then conclusion (1) of the theorem follows from (28) and (29), use of superposition, the above lemma, the last of the assumptions in (4), and boundedness of $\{y_t^r\}$.

(2) Multiplying (9) by $r_p\mathbf{E}^{\mathrm{T}}$, adding $\lambda\Delta u_t^p$ on both sides, and then using (16) we have

$$r_p\mathbf{E}^{\mathrm{T}}\mathbf{y} + \lambda\Delta u_t^p = r_p^2 e\Delta u_t^p + \lambda\Delta u_t^p + r_p e\sum_{i=1}^{p-1} r_i\Delta u_t^i + r_p\mathbf{E}^{\mathrm{T}}\left(\mathbf{G}y_t + \mathbf{H}\sum_{i=1}^{p} r_i\Delta u_{t-1}^i + \mathbf{F}\right) \tag{30}$$

which results in

$$\Delta u_t^p = Py_{t+N_1} + \lambda'\Delta u_t^p - v_{t+N_1} - \alpha y_t - \beta\sum_{i=1}^{p} r_i\Delta u_{t-1}^i - ke\sum_{i=1}^{p-1} r_i\Delta u_t^i \tag{31}$$

where

$$v_{t+N_1} = k\sum_{j=1}^{N_1} e_{j=1}F_j\omega_{r+j} \tag{32}$$

Defining

$$\phi_{t+N_1} = Py_{t+N_1} + \lambda'\Delta u_t^p \tag{33}$$

we can write (31) as

$$\phi_{t+N_1} - v_{t+N_1} = \alpha y_t + \beta\sum_{i=1}^{p} r_i\Delta u_{t-1}^i + \Delta u_t^p + ke\sum_{i=1}^{p-1} r_i\Delta u_t^i \tag{34}$$

Here we note that $\phi_{t+N_1} - v_{t+N1}$ is $\mathscr{F}_t$-measurable. It is obvious that $\phi_{t+N_1} - v_{t+N_1}$ is the optimal linear prediction of ϕ_{t+N_1} given $\mathscr{F}_t$, i.e.,

$$\phi_{t+N_1}^0 = \phi_{t+N_1} - v_{t+N_1} = \alpha y_t + \beta\sum_{i=1}^{p} r_i\Delta u_{t-1}^i + \Delta u_t^p + ke\sum_{i=1}^{p-1} r_i\Delta u_t^i \tag{35}$$

Now, making use of the fact that $\phi_{t+N_1} = \phi_{t+N_1}^0 + v_{t+N_1}$ and then substituting this equation and (33) into (25), we obtain after some manipulations

$$J' = E\{(\phi_{t+N_1}^0 - Py_{t+N_1}^r)^2|\mathscr{F}_t\} + E\{v_{t+N_1}^2|\mathscr{F}_t\} \geqslant E\{v_{t+N_1}^2|\mathscr{F}_t\} \tag{36}$$

The first term in the middle part of (36) is greater than or equal to zero. It becomes equal to zero by putting

$$\phi_{t+N_1}^0 = Py_{t+N_1}^r \tag{37}$$

Substituting (35) onto (37) yields the control law defined by (17). Finally, from (32) we obtain

$$E\{v_{t+N_1}^2|\mathscr{F}_t\} = E\left\{\left(k\sum_{j=1}^{N_1} e_{j-1}\sum_{i=0}^{j-1} f_i\omega_{t+j-i}\right)^2\middle|\mathscr{F}_t\right\} = k^2\sigma^2\sum_{j=1}^{N_1}\left(\sum_{i=0}^{N_1-j} f_i e_{i+j-1}\right)^2 = \gamma^2 \tag{38}$$

(3) Using (6) and the definitions of the polynomials α and β, we can write the polynomial T as

$$T = A\Delta\left(1 - z^{-1}r_p k \sum_{i=1}^{N_1} z^i e_{i-1} E_i\right) + z^{-1} r_p k B \sum_{i=1}^{N_1} z^i e_{i-1} \tag{39}$$

From (39) we find

$$T(1) = r_p k B(1) \sum_{i=1}^{N_1} e_{i-1} = r_p B(1) P(1) \tag{40}$$

Conclusion (3) now follows immediately from (29). □

4. The ANGPC algorithm

We assumed in the previous section that the plant parameters were all known. When the plant parameters are unknown we will have to use a parameter estimator. In this section an adaptive nonlinear generalized predictive control algorithm (ANGPC) is defined by combining the controller derived in Section 2 with a general parameter estimation scheme, Our ANGPC algorithm is based upon the following assumptions:

A1. The polynomial degrees n and m in (1) are known.
A2. p is a known odd positive integer.

We now rewrite the plant equation (1) in the form

$$A' y_t = \sum_{i=1}^{p} B_i \Delta u_{t-1}^i + \omega_t \tag{41}$$

where

$$A' = A\Delta = 1 + a_1' z^{-1} + \ldots + a_{n+1}' z^{-n-1}$$
$$B_i = r_i B = b_0^i + b_1^i + \ldots + b_m^i z^{-m}, i = 1, \ldots, p \tag{42}$$

Computation of the polynomial G_j and two other polynomials E_{ij} and H_{ij} are done from the two following equations

$$1 = F_j A' + z^{-j} G_j \tag{43}$$

$$B_i F_j = E_{ij} + z^{-j} H_{ij} \tag{44}$$

where

$$E_{ij} = e_0^i + e_1^i z^{-1} + \ldots + e_{j-1}^i z^{-j+1}$$
$$H_{ij} = h_0^{ij} + h_1^{ij} z^{-1} + \ldots + h_{m-1}^{ij} z^{-m+1} \tag{45}$$

for $i = 1, \ldots, p$ and $j = 1, \ldots, N_1$.

Define

$$\mathbf{E}_i = [e_0^i\, e_1^i \ldots e_{N_1-1}^i]^{\mathrm{T}}$$
$$\mathbf{H}_i = [H_{i1}\, H_{i2} \ldots H_{iN_1}]^{\mathrm{T}} \tag{46}$$

This allows us to write the control law (17) in the form

$$\mathbf{E}_p^{\mathrm{T}} \sum_{i=1}^{p-1} \mathbf{E}_i \mathbf{u}_i + (\mathbf{E}_p^{\mathrm{T}} \mathbf{E}_p + \lambda) \mathbf{u}_p = \mathbf{E}_p^{\mathrm{T}}\left(\mathbf{y}_r - \mathbf{G} y_t - \sum_{i=1}^{p} \mathbf{H}_i \Delta u_{t-1}^i\right) \tag{47}$$

The ANGPC algorithm now follows below.

(1) The parameters of system (41) are updated from the following estimation scheme (Goodwin *et al.* 1980)

$$\theta_t = \theta_{t-1} + \frac{\rho}{\Lambda_{t-1}} \mathbf{X}_{t-1}[y_t - \mathbf{X}_{t-1}^{\mathrm{T}} \theta_{t-1}],\ \rho > 0 \tag{48}$$

$$\Lambda_t = \Lambda_{t-1} + \mathbf{X}_t^{\mathrm{T}} \mathbf{X}_t,\ \Lambda_0 = 1 \tag{49}$$

where

$$\mathbf{X}_t^{\mathrm{T}} = [y_t \dots y_{t-n} \Delta u_t \dots \Delta u_{t-m} \dots \Delta u_t^p \dots \Delta u_{t-m}^p] \tag{50}$$

$$\theta^{\mathrm{T}} = [-a_1' \dots -a_{n+1}'\ b_0^1 \dots b_m^1 \dots b_p^0 \dots b_m^p] \tag{51}$$

(2) G_j, E_{ij}, and H_{ij} are calculated from (43) and (44), respectively.
(3) The control action u_t is determined from equation (47).

Note that the above parameter estimation scheme is linear, whereas a nonlinear scheme has to be used in the algorithm introduced by Zhu *et al.* (1991) in order to update the parameters of both the linear part and of the nonlinear part, viz. a_i, b_i, and r_i, respectively.

5. Simulation experiments

In order to investigate the performance of the above ANGPC algorithm, we will in this section present some results obtained from simulation experiments. For the purpose of being able to compare results, we will use the same two plants (L1 and L2) and the same two nonlinearities (NL1 and NL2) as were used by Zhu *et al.* (1991). Referring to (1) and (2), the following values are used:

L1: $a_1 = -0{\cdot}9 \quad b_0 = 1 \quad b_1 = 2$
L2: $a_1 = -2{\cdot}87 \quad a_2 = 2{\cdot}74 \quad a_3 = -0{\cdot}87$
$b_0 = 0{\cdot}04 \quad b_1 = 0{\cdot}002 \quad b_2 = -0{\cdot}037$

NL1: $r_0 = 1 \quad r_1 = 1 \quad r_2 = 1$
$r_3 = 0{\cdot}2$
NL2: $r_0 = 0 \quad r_1 = 1 \quad r_2 = 0$
$r_3 = -1$

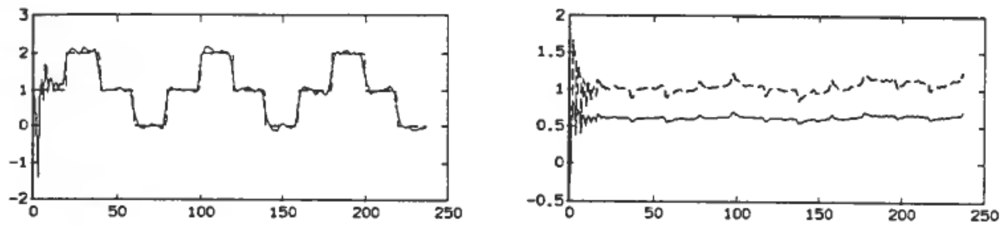
Here we note that L1 is open-loop stable and nonminimum phase, whereas L2 is open-loop unstable and minimum phase. In addition, ω_t is here a zero-mean random disturbance with covariance $\sigma^2 = 0{\cdot}1$.

In order to consider transient behaviour, we assign a set-point sequence as follows

L1:		L2:	
Samples:	Set-point value:	Samples:	Set-point value:
01–20	1	01–40	1
21–40	2	41–80	2
41–60	1	081–120	1
61–80	0	121–160	0

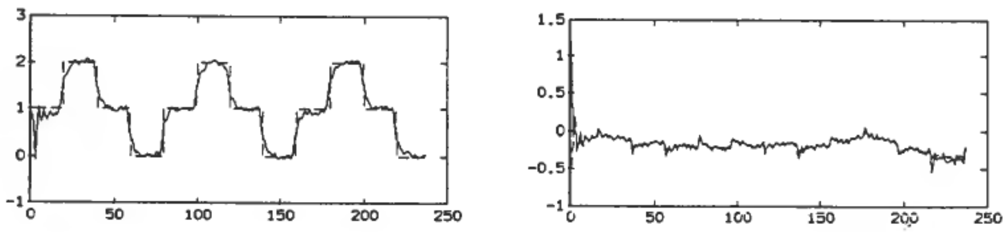
The cycle from respectively 1 to 80 or from 1 to 160 is repeated periodically in each experiment. In the plots shown in Fig. 1–4 the output y_t and the control input u_t are shown as unbroken lines, whereas the set-point sequence y_t^r and the intermediate variable x_t are shown as broken lines.

The parameters of the ANGPC algorithm are chosen as $N_1 = 3$ whereas $\lambda = 0{\cdot}01$ for L1 and $\lambda = 0$ for L2. From the plots in the figures it is seen that the output tracks the set-point sequence quite well even though there is a random disturbance. The large input and output deviations at the outset are more or less what should be expected in a commissioning period when the parameter estimates have not yet converged. The predictive nature of the controller can clearly be seen in the plots, where prior knowledge of a change in the set-point value has caused the output y_t to start moving before the actual change in the set-point has occurred. If we compare with the simulation experiments in Zhu *et al.* (1991), a somewhat better performance of our algorithm is seen. Whereas rapid changes in the control signal u_t occur in the simulation experiments of L2 + NL1 and L2 + NL2 in the above reference, the control signal in our simulation experiments of the same systems (Figs. 3 (*b*) and 4 (*b*) appears to be quite smooth.



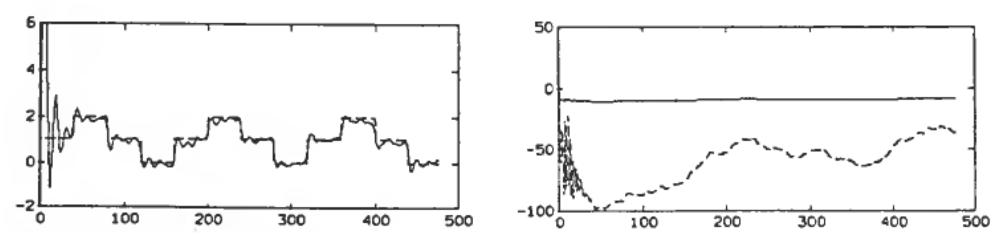
(*a*) (*b*)

Figure 1 (*a*). Output and set-point of system L1 + NL1. (*b*) Control and intermediate variable of L1 + NL1.



(*a*) (*b*)

Figure 2 (*a*). Output and set-point for system L1 + NL2. (*b*) Control and intermediate variable of L1 + NL2.



(*a*) (*b*)

Figure 3 (*a*). Output and set-point for system L2 + NL1. (*b*) Control and intermediate variable of L2 + NL1.

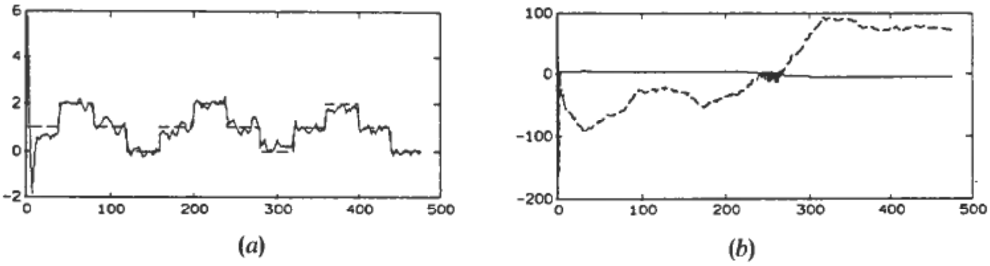


Figure 4 (*a*). Output and set-point of system L2 + NL2. (*b*) Control and intermediate variable of L2 + NL2.

6. Conclusion

In this paper we have derived a nonlinear generalized predictive control algorithm for systems which can be modelled by a Hammerstein model. Stability of the algorithm has, subject to fairly to weak assumptions, been shown with the control horizon $N_u = 1$. We have also suggested an adaptive nonlinear generalized predictive control algorithm which turned out to perform quite well in simulation experiments. Stability and convergence of the latter algorithm have, however, not been shown. As point of fact, we have so far not even tried to do that because analysis of stability and convergence of nonlinear generalized predictive control algorithms in adaptive or self-tuning form is indeed a very difficult task to carry out.

APPENDIX

Proof of the Lemma

Using superposition and Lemmas A.1. and A.5 in Goodwin *et al.* (1981), we obtain

$$\frac{1}{N}\sum_{t=1}^{N}(\Delta u_t^p)^2 \leqslant \frac{L_1}{N}\sum_{t=1}^{N} y_{t+d}^2 + \sum_{j=1}^{p-1}\frac{L_{2j}}{N}\sum_{t=1}^{N}(\Delta u_t^j)^2 + \frac{L_3}{N} \text{ wp. } 1 \tag{A. 1}$$

Define, for $j = 1, 2, \ldots, p$,

$$\delta_t^j = \begin{cases} \Delta u_t^j \text{ if } |\Delta u_t^j| > M,\ M > 0 \\ 0 \text{ otherwise} \end{cases} \tag{A. 2}$$

We then have, for $j = 1, 2, \ldots, p$,

$$\frac{1}{N}\sum_{t=1}^{N}(\Delta u_t^j)^2 \leqslant \frac{1}{N}\sum_{t=1}^{N}(\delta_t^j)^2 + M^2 \tag{A. 3}$$

Take M large enough such that

$$\frac{L_{2j}}{N}\sum_{t=1}^{N}(\delta_t^j)^2 \leqslant \frac{1}{2(p-1)N}\sum_{t=1}^{N}(\Delta u_t^p)^2 \tag{A. 4}$$

It then follows that

$$\frac{1}{N}\sum_{t=1}^{N}(\Delta u_t^p)^2 \leqslant \frac{L_1}{N}\sum_{t=1}^{N} y_{t+d}^2 + \frac{1}{2N}\sum_{t=1}^{N}(\Delta u_t^p)^2 + L_4 \text{ wp. } 1 \tag{A. 5}$$

and hence

$$\frac{1}{N}\sum_{t=1}^{N}(\Delta u_t^p)^2 \leqslant \frac{K_1}{N}\sum_{t=1}^{N} y_{t+d}^2 + K_2 \text{ w.p. } 1 \tag{A. 6}$$

□

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