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Sophus Lie

NILS A. BAAS†

In 1992 it was 150 years since the mathematician Sophus Lie was born. It is a great pleasure to see that today on Founder's day The Royal Norwegian Academy of Sciences and Letters honours him by having his portrait on this year's memorial medal.

It is a difficult task to give an account of the work of a mathematician in general terms. Permit me to quote from the memorial speech made by L. Sylow on Lie: 'It is the misfortune of the mathematician, more than any other scientist, that his work cannot be presented and explained to the well-educated public, not even to a general audience of scientists. One has to be a mathematician in order to sense the special beauty which a mathematical theorem can offer, or to admire the pure lines of the finished parts of this science.'

In spite of this quotation I will try to give you an impression of the life and work of Sophus Lie.

Marius Sophus Lie was born in Nordfjordeid in the western part of Norway on December 17, 1842. His father, Johan Herman Lie, was a minister, and his mother came from a well-known Trondheim family. At an early age the family moved to Moss (near Oslo), where he grew up. Later on he was sent to Nissens Latin- og Realskole in Christiania (Oslo) where he graduated in 1859.

At school Lie did well in all subjects, he was a generally well-gifted pupil. In Mathematics he had Ludvig Sylow, later on a well-known mathematician, as a teacher. Sylow remarks later that even if Lie was ahead of his class in mathematics, he did not

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† Department of Mathematical Sciences, NTH, University of Trondheim. N-7034 Trondheim, Norway.

see a great mathematician in him. Lie himself expressed that the road to mathematics for him was long and difficult.

Sophus Lie was extremely well equipped, not only intellectually, but also physically. He was tall and strong and did very well in sports. It was therefore natural for him to think of a military career, but his astigmatism stopped him. He considered studying both languages and science, and his friends thought that a bad mark in Greek made Lie switch to science.

During his studies Lie attended lectures by Sylow on group theory, a subject not being lectured at many places at that time and which appeared to have fundamental importance for his later research.

He graduated from the University at Christiania in 1865. He did not graduate with distinction as he had hoped for, and he was very disappointed. The years just after graduation were very difficult—because he did not know what to do. His talents were so many, and he could not make up his mind. He was teaching mathematics, working as an assistant in astronomy, but without finding any satisfaction.

The turning point in Lie's life came in 1868. He came across papers by J. V. Poncelet and J. Plücker and was introduced to what was then modern geometry. Suddenly he felt that he had found a field where he could unfold his talent—especially the ability to think geometrically. Lie seemed excited by Plücker's idea of replacing the point as volume element by more complicated objects like lines, curves and surfaces. From now on he was fully devoted to mathematics, and he seemed happy with his choice.

He soon felt the need to communicate his results, but formal publications were too tedious for him. Therefore in this early period he made use of small pamphlets containing isolated theorems without proofs and in a non-edited form. To Lie it seemed important to secure as soon as possible the priority of the scientific discoveries he made.

His first published paper appeared in 1869. It gives a new representation of the complex plane and uses ideas of Plücker. But Lie had difficulties in getting these ideas published by the Academy in Christiania. He was impatient. Professor Bjerknes asked for more time to look at the paper, but Professor Broch returned it after two days—saying he had understood nothing!

However, three other professors, who probably understood the material even less, supported publication. This happened as a result of influence by friends of Lie. The important consequence of this was that Lie now received a travel grant which made it possible for him to go abroad.

Elling Holst writes in a paper in 1912 that many stories were told in Christiania at the time about Lie and his physical strength and good condition. One weekend he was going to visit his parents in Moss. He walked the 60 km. When he did not find his parents at home, he immediately turned around and walked the same way back! He was also known as an excellent gymnast.

Lie spent the winter 1869–70 in Berlin. Here he met another young gifted geometer, Felix Klein. They became friends and were to influence each other strongly in the following years. He also received positive attention among the leading mathematicians in Berlin. From Berlin he went to Paris via Göttingen, later on Klein also came to Paris. Here in Paris Lie started to work on transformation groups which was to become one of his major fields of research.

Because of the French–German war in 1870–71 Klein had to return to Germany. Lie decided to walk to Italy. However, he was soon arrested, accused of being a German spy. His mathematical notes were assumed to be military secrets in code, especially a letter from Klein seemed suspicious! After intervention by the French mathematician

G. Darboux, Lie was released. Thereafter he travelled back to Christiania through Switzerland and Germany without further complications.

At home he received a fellowship from the University, and in 1871 he received his doctor's degree defending his dissertation entitled: 'Over en Classe geometriske Transformationer'. Lie now applied for a professorship in Lund (Sweden), but many of his influential friends appealed to the Parliament (Stortinget), and consequently he was appointed to an extraordinary professorship in Christiania in 1872.

In 1874 he married Anna Birch from Tvedestrand, they had been engaged since 1872. They had three children, two girls and one boy. The marriage was a very happy one, and Lie was very fond of his family.

Lie entered a very rich and productive scientific period. His starting point had been geometry, but now he turned to the theory of differential equations with all his forces. He began to see the great connections between his transformation groups and general symmetries. Lie was still in contact with Klein who published his Erlangen program in 1872. Here Klein defines geometry as the study of those properties of a space which are left invariant under a group of transformations.

Back to the notion of symmetry. In Norway it is well known that N. H. Abel showed the impossibility of solving the general quintic equation or higher by radical expressions. However, the full understanding of these questions first came through the work of the French mathematician E. Galois who studied the symmetries of these equations.

Lie discovered that his theory of transformation groups could be used to give a corresponding theory of differential equations. Working out the theory took a long time, and it was not until 1893 that 'Theorie der Transformationsgruppen' was finished in three volumes. In Christiania Lie was mathematically isolated, and he complained to Klein. In 1884 Klein sent one of his students, Friedrich Engel, to Christiania to study with Lie. They developed a warm and lifelong friendship. Engel helped Lie by giving his often intuitive geometrical ideas a more precise mathematical form. Lie often felt it a burden to give his ideas a form suitable for publication. Despite his isolation the 1870's were the most intensive and creative years of his life.

At the same time he was also disappointed that his works did not receive more attention abroad. In a letter to the German mathematician A. Mayer he writes: 'If I only knew how to get the mathematicians interested in transformation groups and their applications to differential equations. I am certain, absolutely certain in my case, that these theories in the future will be recognized as fundamental. I want to form such an impression now, since for one thing, I could then achieve ten times as much.'

This disappointment led him in 1876 to start working on differential geometric subjects again. In the fall of 1882 Lie went to Paris, and he seemed to have impressed the leading French mathematicians. From now on they followed his work with great interest.

In 1886 Lie was offered a professorship in Leipzig as Klein's successor, as the latter having moved to Göttingen. The offer was very tempting for Lie, and he accepted. Here he could come to a great mathematical centre where he could lecture on his own research. But he did not want to stay for ever, he had in mind 6–8 years abroad. So he did not resign from his professorship in Christiania, but he was granted leave of absence in an extraordinary way.

For Lie it was not vital to get his ideas published. Therefore it was also important for him that in Leipzig he was close to the printers of the extensive production he was planning. His great work on transformation groups with Engel was published in the period 1888–93.

At this time it was quite unusual for young French mathematicians to go to Germany to study. But Lie found that the Ecole Normale Superieure in Paris sent some of its best students to him, and he was very proud of this.

Life in Leipzig was not that easy for Lie. His teaching duties were much heavier than at home, the language caused him some problems, and he got tired of supervising weak and dependent graduate students. As time passed he also ran into trouble with some of his colleagues.

In the fall of 1889 he suffered a severe case of neurasthenia and insomnia. He entered a psychiatric clinic near Hannover and describes himself as an impossible patient, resisting the opium cure he was subjected to. He got over this crisis and started to teach again in the fall of 1890. But from his letters it seems he did not get well until 1892. Then he writes that he has regained his old health and 'with sleep, the pleasure of life and work has returned'.

Lie was missing Norway and longing to be back. He found the nature around Leipzig boring and wrote: 'Yes, I cannot find words to express how much I am longing back to Norway. My nervous system has suffered a lot here in Leipzig where I have missed the opportunity for exercise and the spiritual influence by nature'.

After the breakdown it seems that he regained his scientific power, but as a human being he seemed more depressed and pessimistic. At the same time he also entered into sharp polemic attacks against colleagues. Many myths have been created because of Lie's illness, and may be it has been used to 'explain' too many things.

In 1892 there was a conflict between Klein and Lie. The Erlangen program of Klein from 1872 had not been as influential as the work of Lie. Klein wanted to republish it with an addition involving the ideas of Lie, but they had very different points of view on how things had developed. In addition Klein had burnt all the letters from Lie to 1877—contrary to an agreement they had. The old friends had fallen out.

Furthermore in 1893 Lie shocked the German mathematicians by publicly attacking Klein, who now had a leading position in German mathematics. He writes: 'I am no pupil of Klein, nor is the opposite the case, although this might be closer to the truth'.

The famous Norwegian author Bjørnstjerne Bjørnson was told by Fridtjof Nansen that Lie was longing back to Norway, and he took the initiative to get him home. In 1894 the Parliament established an honorary chair in the theory of transformation groups and with a special salary, so that he could return without any financial loss. But it was not until 1898 that he left Leipzig and moved back to Christiania.

Lie was a sick man when he came home. He started lecturing in the fall of 1898, but he had to stop after a few months. He suffered from pernicious anaemia, and the illness progressed quickly. Sophus Lie died on February 18, 1899.

The contributions to science by Sophus Lie are very impressive. He created fundamentally new theories and fields of research which have greatly influenced the mathematical development in our century. Lie groups and Lie algebras are today classical notions in mathematics. Today it seems no exaggeration to claim that Sophus Lie is one of the greatest and most influential names in the history of mathematics.

But also in other sciences where fundamental symmetries play a role, as in physics and mechanics, the work of Lie has had an enormous influence. Especially in elementary particle physics Lie groups have played an essential role. The most modern theories of elementary particles, gauge theories and string theories, could not even have been formulated without the work of Lie. Everything seems to indicate that the ideas of Lie will continue to influence the development of mathematics and science in the future.

Lie himself was convinced about this and in a letter to his friend E. Motzfeldt in 1890 he writes: '... my life's work will stand through all times, and in the years to come, be more and more appreciated—no doubt about it'. He was absolutely right!

Finally I regret that a proper biography on Sophus Lie has not yet been written. Many myths have been created about him, many because of his illness. Many of those he was in conflict with survived him, and they may have been too influential in forming our impression of Lie. It is therefore important to study his correspondence and other available material carefully in order to get a better understanding of his personality.

Let us hope that in connection with the 150th anniversary of his birth, an initiative will be taken to give him the biography he truly deserves.

At the University of Oslo 'The Sophus Lie Memorial Week' took place August 17–21, 1992, including an exhibition of his life and work.

A bust of Sophus Lie was unveiled at his birthplace Nordfjordied in Norway on August 22, 1992.

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SOURCES

ENGEL, F. (1992). Sophus Lie. *Norsk Mat. Tidsskrift*, **4,** 97–114.

HOLST, E. (1899). Speech at 'Universitetes Mindefest over Sophus Lie', April 20, 1899, published by Aftenposten, Kristiania.

HOLST, E. (1899). Træk af Sophus Lies ungdomsliv. *Ringeren*, **2,** 1899, 98–102, Kristiania.

HOLST, E. (1912). Sophus Lie. in *Nordmænd i det nittende Aarhundre*, **2,** 1912, 99–143, Kristiania.

JOHANSSON, I. (1942). Sophus Lie—betraktininger i anledning av hundreårsdagen for hans fødsel, *Norsk Mat. Tidsskrift*, **24,** 97–106.

JOHANSSON, I. (1949). Sophus Lie—et femtiårsminne, article in *Aftenposten*, Oslo, February 18, 1949.

SYLOW, L. (1900). Sophus Lie—en Mindetale, *Kristiania Videnskabsselskab*, February 24, 1899, published 1900, Kristiania.

STRAUME, E. (1983). *Sophus Lie i historisk perspektiv.* Inst. for Mat. Realfag, Univ. of Tromsø, 1983.