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Estimation of regurgitant volume and orifice in aortic regurgitation combining c.w. Doppler and parameter estimation in a Windkessel-like model†

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A method for noninvasive estimation of regurgitant orifice and volume in aortic regurgitation is proposed and tested in anaesthesized open chested pigs. The method can be used with noninvasive measurement of regurgitant jet velocity with continuous wave ultrasound Doppler measurements together with cuff measurements of systolic and diastolic systemic pressure in the arm. These measurements are then used for parameter estimation in a Windkessel-like model which include the regurgitant orifice as a parameter. The aortic volume compliance and the peripheral resistance are also included as parameters and estimated in the same process. For the test of the method, invasive measurements in the open chest pigs are used. Electromagnetic flow measurements in the ascending aorta and pulmonary artery are used for control, and a correlation between regurgitant volume obtained from parameter estimation and electromagnetic flow measurements of 0·95 over a range from 2·1 to 17·8 mL is obtained.

1. Introduction

The blood velocity in an aortic regurgitation jet v can be measured noninvasively using continuous wave ultrasonic Doppler techniques. The drive pressure of the jet, which is the difference between the aortic pressure p_a and the left ventricular pressure p_v, can approximately be estimated from the Bernoulli equation (Hatle and Angelsen 1985):

$$p_a - p_v = 4v^2 \tag{1}$$

Noninvasive quantitation of the degree of aortic regurgitation based on the diastolic decay rate of this pressure difference or the jet velocity has been proposed by many authors (Labovitz *et al.* 1986, Mauyama *et al.* 1986, Teague *et al.* 1986). This gives a noninvasive estimation of the degree of regurgitation, but the relationship between the regurgitant volume and the fall rate of this pressure difference is also influenced by other factors such as arterial compliance and total peripheral resistance (Sloerdahl *et al.* 1987).

The diastolic decay rate of the aortic pressure is in the normal situation caused by outflow of blood to the peripheral organs from the compliant aortic reservoir. With aortic regurgitation there is an additional outflow of blood from the aorta back to the

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left ventricle, causing an increased diastolic decay rate of the pressure. Thus, the diastolic decay rate of the pressure is an indication of the degree of the regurgitation, but the aortic compliance and the peripheral resistance plays an important role in this pressure fall, causing uncertainty in the earlier presented methods. Also, with high degree of aortic regurgitation, the back flow of blood into the left ventricle will give increased end diastolic left ventricular pressure, giving an additional decrease in the jet velocity.

The present paper uses parameter estimation in a Windkessel-like model to obtain estimates of the regurgitant orifice and volume. The aortic volume compliance and the peripheral resistance are also estimated. The measurements used for the parameter estimation are the systolic and diastolic pressures in the aorta together with noninvasive Doppler measurements of systolic outflow from the left ventricle as well as the maximal velocity in the regurgitant jet. The systolic and diastolic pressures in the aorta can for instance be obtained with cuff measurements, so that all measurements can be done noninvasively.

An analysis of the model and estimation scheme is given below. The estimation scheme is tested in open chested pigs where aortic regurgitation is generated with a cage of metal wires at the tip of a catheter, and electromagnetic flow measurement in the ascending aorta and pulmonary artery are used for control. A correlation factor of 0·95 is found between the regurgitant volume determined from the regurgitant jet velocity as measured with this method compared to electromagnetic measurement of blood velocity.

2. The model

In systole the Windkessel-like model shown in Fig. 1 is used to represent the relationship between flow from the left ventricle into the aorta. The analog electrical circuit used is taken from Goldwyn and Watt (1967). The voltage source represents the left ventricle producing a pressure p_v. The flow of blood from the ventricle into the aorta is q_a, and the pressure in the aorta is p_a. The aorta is modeled as a compliant reservoir of blood with volume compliance C, and the peripheral organs are modeled by a peripheral resistance R with a flow rate of blood from the aorta to the periphery of q_p. The serial inductance represents the inertia of the blood as discussed in Goldwyn and Watt (1967).

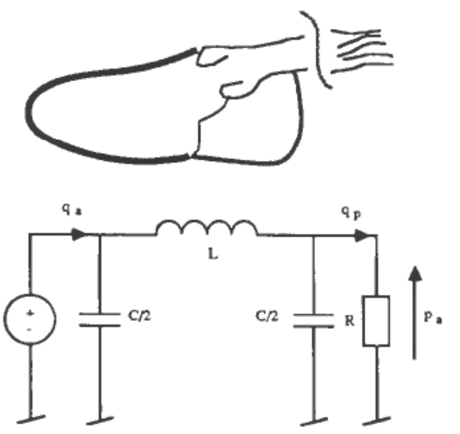

Figure 1. Systolic model for the left ventricle and the aorta with peripheral circulation.

The model is described by the following differential equation

$$\frac{dp_a}{dt}+\frac{1}{T}p_a+\frac{L}{2R}\frac{d^2p_a}{dt^2}+\frac{LC}{4}\frac{d^3p_a}{dt^3}=\frac{1}{C}\left\{q+\frac{L}{R}\frac{dq}{dt}+\frac{LC}{2}\frac{d^2q}{dt^2}\right\} \tag{2}$$

where $T=RC$ is the time constant of the aortic to peripheral discharge.

In diastole the model is modified as in Fig. 2 where we now have a discharge of blood from the aortic reservoir both into the peripheral circulation and the left ventricle through the leaking valve. Since the acceleration of the flow is low, the inductance in the electrical analog model can be neglected. A highly accelerated flow as that through the leaking valve will give a flat velocity profile in the leaking hole, i.e., the blood velocities are the same across the hole (or orifice) (Hatle and Angelsen 1985). The rate of volume flow is then the product of this velocity v and the area A of the hole which gives

$$q_r=Av=1/2A\{p_a-p_v\}^{1/2} \tag{3}$$

The volume flow is related to the pressure difference p_a-p_v through (1). The regurgitant flow is represented by a nonlinear resistance in the electrical analog model.

The ventricular pressure p_v is modeled as a voltage source generating a back pressure towards the regurgitant flow as the ventricle is filling during diastole, both from the normal mitral inflow, and the pathological regurgitant flow. The rate of change of the aortic pressure is

$$\frac{dp_a}{dt}=\frac{dp_a}{dV_a}\{-q_r-q_p\}=-\frac{1}{C}\{q_r+q_p\} \tag{4}$$

where V_a is the volume of the arterial system.

Rearranging (1) we get $p_a=4v^2+p_v$. The peripheral flow is the aortic pressure divided by the peripheral resistance $q_p=p_a/R$ and the regurgitation flow is given in (3). Inserting these expressions into (4) and rearranging somewhat, we obtain a differential equation for the regurgitant jet velocity in diastole

$$\frac{dv}{dt}=-\frac{1}{2T}v-\frac{1}{8T}\left(p_v+T\frac{dp_v}{dt}\right)\frac{1}{v}-\frac{A}{8C} \tag{5}$$

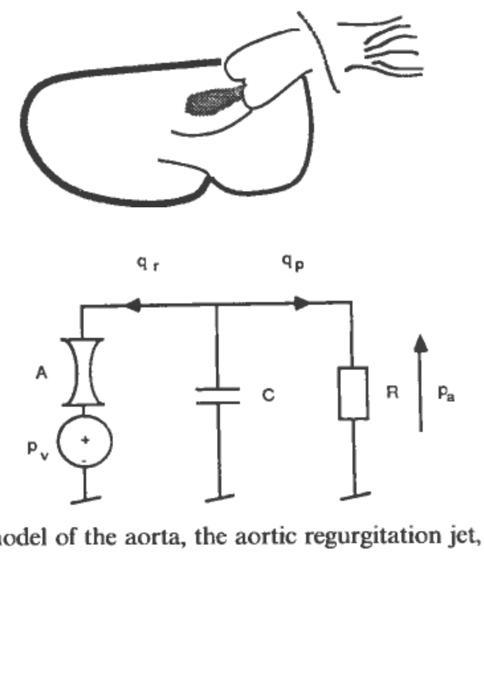

Figure 2. Diastolic model of the aorta, the aortic regurgitation jet, and the left ventricle.

The equation is a nonlinear differential equation for the regurgitant velocity, and the regurgitant velocity can be measured noninvasively with ultrasonic Doppler techniques. It contains three parameters: T, C, and A, and an unknown function $p_v(t)$.

The goal is to estimate the parameters, especially the regurgitant orifice, because when this is known the regurgitant volume flow rate can be calculated using (3), and by integrating the flow rate over the whole diastole, the regurgitant volume is calculated.

3. Estimation scheme

Noninvasive ultrasonic Doppler measurement of the blood velocity in the regurgitant orifice is the basis for the parameter estimation. The velocity in the orifice can be obtained as the maximal velocity in the Doppler spectrum in continuous wave Doppler measurements of the regurgitant jet. The observability for all three independent parameters T, C, and A using this maximal velocity and (5), however, is zero. Therefore, more measurements, or an improved model, has to be added.

In the first half of the diastole there is usually no extra increase in the LV pressure from the regurgitation. The LV pressure is low and, to a first approximation, can be set to zero. It normally starts close to zero in early diastole and increases with some oscillations caused by flow acceleration at the opening of the mitral valve and inertia in the flow column from the pulmonary veins through the left atrium and mitral valve into the left ventricle. Samstad *et al.* (1989) has developed a method based on measurement of velocity acceleration in the mitral inflow to estimate the pressure in the left ventricle when the mitral flow is zero. According to this method the LV pressure when the mitral flow is zero is

$$p_v(T_0) = 0{\cdot}6[dv_m/dt]_{\max} \tag{6}$$

where T_0 is the time of zero mitral flow in diastole, and v_m is the blood velocity through the mitral valve. As a second approximation of $p_v(t)$ in (6) we can neglect the oscillations and approximate the left ventricular pressure by a linearly increasing function throughout diastole

$$p_v(t) = Kt \tag{7}$$

where $t = 0$ at the beginning of diastole, and K is determined so that (6) is satisfied

$$K = 0{\cdot}6[dv_m/dt]_{\max}/T_0. \tag{8}$$

Even in large insufficiencies there is no extra increase in the LV pressure before the latter half of diastole, so if we use this scheme during the first half diastole for parameter estimation (7) is a good approximation.

In order to make the system observable still, other relations have to be found between the parameters T, C, and A. This can be done by measuring the volume flow of blood into the aorta during systole as well as measuring the systolic and diastolic blood pressures with cuff. The systolic volume flow can be measured using noninvasive ultrasonic techniques: the velocity in the aortic annulus is measured by ultrasonic Doppler techniques and multiplying this velocity with the area of the annulus measured by two dimensional ultrasonic imaging, the volume flow can be obtained (Samstad *et al.* 1989, Skjaerpe *et al.* 1985).

Assuming $t=0$ at start of systole we obtain the following boundary conditions:

$$p_a(0)=P_d \qquad \text{Measured with cuff} \tag{9}$$

$$p_{a\max}=P_s \qquad \text{Measured with cuff} \tag{10}$$

$$p_a(T_s)=P_c=4v_{\max}^2 \qquad \text{Measured from regurgitant jet velocity} \tag{11}$$

where T_s is the duration of the systole, P_d and P_s are the diastolic and systolic pressures measured with cuff, and P_c is the aortic pressure at the closure of the aortic valve. This is equal to the early diastolic pressure drops across the valve since the early diastolic ventricular pressure can be approximated to zero.

To find relations between R, C, and A we integrate (2) over systole assuming that the derivatives of both p_a and q can be approximated to zero at the beginning and the end of the systole. We then get

$$C[P_c-\bar{P}_d]+P_sT_s/R=Q_s \tag{12}$$

where $\bar{P}_s$ is the average aortic pressure during systole. From this equation we can express the compliance C as a function of the peripheral resistance R

$$C=\frac{Q_s-\bar{P}_sT_s/R}{P_c-P_d} \tag{13}$$

The systolic stroke volume is calculated as

$$Q_s=A_s\int_0^{T_s} v_s(t)\,dt \tag{14}$$

where v_s is the systolic velocity through the aortic annulus, and A_s is the systolic area of the aortic annulus. The flow rate into the aorta then becomes $q=A_sv_s$. The diastolic regurgitant volume is obtained as

$$Q_r=ADI \tag{15}$$

where

$$DI=\int_0^{T_d} v(t)\,dt \tag{16}$$

where $v(t)$ is the diastolic jet velocity and DI is the diastolic jet velocity integral where we have defined $t=0$ at the beginning of the diastole and T_d is the duration of the diastole. The net stroke volume then becomes

$$Q_n=Q_s-Q_r=Q_s-ADI \tag{17}$$

By definition of the peripheral resistance, the net stroke volume is equal to

$$Q_n=\bar{P}(T_s+T_d)/R \tag{18}$$

where $\bar{P}$ is the average aortic pressure over the cardiac cycle, and T_s+T_d is the length of the cardiac cycle. Combining (17) and (18), we obtain

$$R=\frac{\bar{P}(T_s+T_d)}{Q_s-ADI} \tag{19}$$

The mean systolic pressure and aortic mean pressures are approximately estimated from the diastolic, systolic, and closure pressures as

$$\bar{P}_s = P_d + 0{\cdot}7(P_c - P_d) \tag{20}$$

$$\bar{P} = (2P_d + P_s)/3. \tag{21}$$

Using (13), (19), (20), and (21) we can through non-invasively measured variables relate both C and $T = RC$ to A, and (5) essentially contains one independent parameter, the regurgitant orifice A. We can then use parameter estimation techniques to estimate this parameter based on CW Doppler measurements of the maximal velocity in the regurgitant jet. Once the regurgitant orifice is found, the regurgitant volume is calculated from (15) and (16) and with the stroke volume from (14) the regurgitant fraction can be found as

$$\mathrm{RF} = Q_r/Q_s \tag{22}$$

For parameter estimation we have used the augmented Kalman filter (Jazwinsky 1970).

This method is based on defining a state-space model of the system which can be done on the basis of (5) defining the jet velocity as one state variable x_1 and relating T and C to the regurgitant area A using (13), (19), (20), and (21). The regurgitant orifice A is defined as a second state variable x_2 and the jet velocity v is the only variable measured.

We also change the continuous time model to a time discrete model since the calculations are done by a digital computer. Using a sampling period in the temporal domain Ω we approximate

$$t = k\Omega$$

$$\boldsymbol{x}'(t) = \frac{\boldsymbol{x}(k+1) - \boldsymbol{x}(k)}{\Omega} \tag{23}$$

giving

$$x_1(k+1) = \left[1 - \frac{\Omega}{2T}\right]x_1(k) - \Omega\frac{K(k\Omega + T)}{8T}\frac{1}{x_1(k)} - \frac{\Omega}{8C}x_2(k) + v_1(k)$$

$$x_2(k+1) = x_2(k) + v_2(k) \tag{24}$$

where the vector $\boldsymbol{v}(k) = \{v_1(k), v_2(k)\}$ is a white noise source representing unknown inputs and errors in the model. The above equation can be written in the compressed form, defining the vector function $\boldsymbol{F}$

$$\boldsymbol{x}(k+1) = \boldsymbol{F}\{\boldsymbol{x}(k), \boldsymbol{v}(k)\} \tag{25}$$

The jet velocity is the only state variable measured

$$y(k) = D\boldsymbol{x}(k) + w(k) = x_1(k) + w(k)$$

$$D = [1, 0] \tag{26}$$

The process noise $\boldsymbol{v}(k) = [v_1(k), v_2(k)]$ and the measurement noise $w(k)$ are assumed to be white noise sequences which has no mutual correlation

$$E\{\boldsymbol{v}^{\mathrm{T}}(k)\boldsymbol{v}(1)\} = \begin{pmatrix} V_1 & 0 \\ 0 & V_2 \end{pmatrix}\delta_{k1}$$

$$E\{w(k)w(1)\} = W\,\delta_{k1} \tag{27}$$

The Kalman filter starts with an *a priori* estimate of the state $\boldsymbol{x}(k)$ and modifying this based on the measurement to the final and *a posteriori* estimate $\boldsymbol{x}(k)$. This modification is based on the formulas

$$\begin{aligned} \boldsymbol{x}(k) &= \boldsymbol{x}(k) + K(k)[y(k) - D\boldsymbol{x}(k)] \\ K(k) &= XD^{\mathrm{T}}[DXD^{\mathrm{T}} + W]^{-1} \end{aligned} \tag{28}$$

where $K(k)$ is the gain of the Kalman filter, and X is the covariance matrix of the *a priori* state estimate. Calculation of these quantities are obtained from the formulas

$$\begin{aligned} \boldsymbol{x}(k+1) &= F\{\boldsymbol{x}(k), \boldsymbol{v}(k), k\} \\ X(k+1) &= \Phi(k)X(k)\Phi^{\mathrm{T}}(k) + V(k) \\ X(k+1) &= \{I - K(k+1)D\}X(k+1) \end{aligned} \tag{29}$$

where we have defined

$$\Phi(k) = \left.\frac{\partial \boldsymbol{F}\{\boldsymbol{x}(k)\}}{\partial x(k)}\right|_{x(k)=x(k)} \tag{30}$$

For the estimation a single cardiac cycle is used. The diastolic data were run through the filter 40 times, starting the first time with the following initial conditions

$$\begin{aligned} \boldsymbol{x}(0) &= [6\,\mathrm{m/s},\ 10\,\mathrm{mm}^2] \\ X_0 &= \begin{pmatrix} 2{\cdot}5 & 0 \\ 0 & 5 \end{pmatrix} \end{aligned} \tag{31}$$

and for every new pass through the data, the *a posteriori* state and covariance matrix from the last datapoint in diastole were used as initial conditions for the next path.

Good convergence of the filter was established on experimental basis using a covariance matrix for the process noise of

$$V = \begin{pmatrix} 10^{-3} & 0 \\ 0 & 10^{-2} \end{pmatrix} \tag{32}$$

and for the measurement noise

$$W = 10^{-5} \tag{33}$$

4. Materials and methods

The method has been tested in three thoracotomized pigs that were anaesthetized with pentobarbital. Aortic regurgitation was produced by opening a cage basket catheter inserted into the aortic valve. Electromagnetic flow probes were placed snugly around the ascending aorta immediately above the aortic valve, and around the pulmonary artery right after the valve. Regurgitant aortic flow was measured as the difference between systolic aortic and pulmonary flow. Before producing regurgitation mean blood flow through the ascending aorta and the pulmonary artery were recorded and calibrated to identity. The pressures in the aorta and the left ventricle were obtained via fluid filled catheters connected to Statham P23ID transducers.

In this first experiment we are interested in the basic validity of the model and the parameter estimation scheme for estimating regurgitant orifice. To minimize errors in the measurements we have used Q_s obtained from electromagnetic measurements, and P_s and P_d obtained from the catheter measurements of the pressure. Ultrasonic measurements are only used to obtain the diastolic jet velocity. The closure pressure P_c

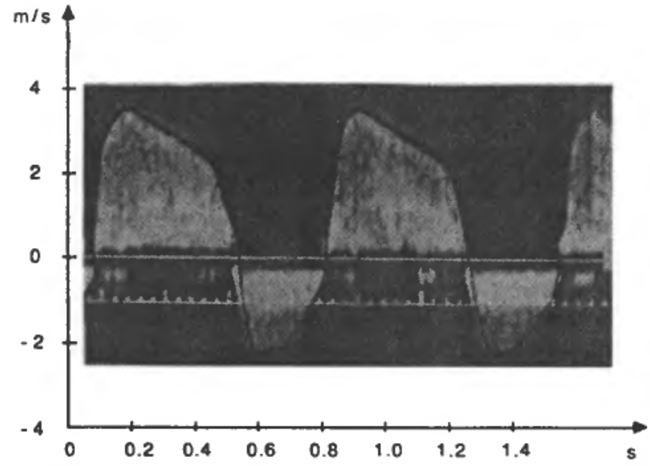

Figure 3. Typical CW Doppler spectrum of systolic LV outflow and the diastolic regurgitant jet obtained by placing the probe at the apex of the heart.

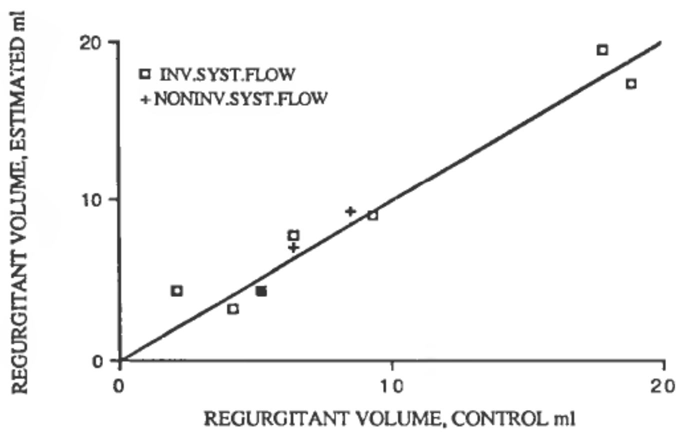

Figure 4. Regurgitant volume obtained by parameter estimation, vertical axis, versus measured with electromagnetic flow probes. Systolic flow were measured with the electromagnetic flow probes in these experiments besides the crosses where the systolic flow was obtained by integrating the blood velocity in the LV outflow tract as obtained with pulsed Doppler.

as obtained from the catheter was compared with the value found from the Doppler measurements of the diastolic jet velocity using (9) and the two values were practically equal.

Figure 3 shows a typical CW Doppler blood velocity recording from the apex of the heart of systolic LV outflow and the diastolic regurgitant jet velocity. The diastolic jet velocity $v(t)$ is found as the maximum velocity in the diastolic Doppler spectrum. This is entered into the computer and used for parameter estimation, by tracing the edge of the spectrum on a digitizing tablet.

5. Results

The results of the parameter estimation are shown in Fig. 4 which shows on the vertical axis the regurgitant volume obtained from parameter estimation of the regurgitant orifice and (13). On the horizontal axis is shown the regurgitant volume

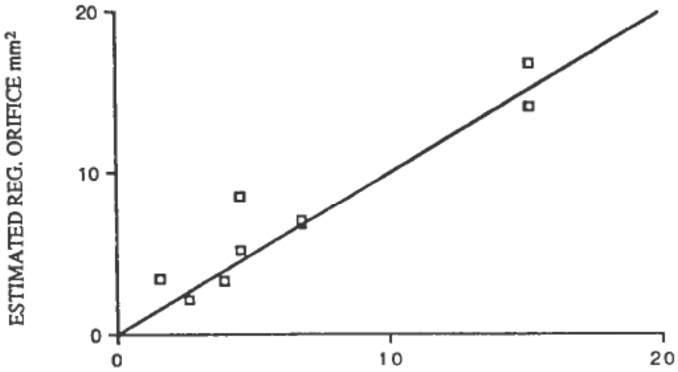

Figure 5. Regurgitant orifice obtained by parameter estimation, vertical axis, versus obtained from dividing the regurgitant volume obtained with the electromagnetic flow probes by the diastolic velocity integral DI.

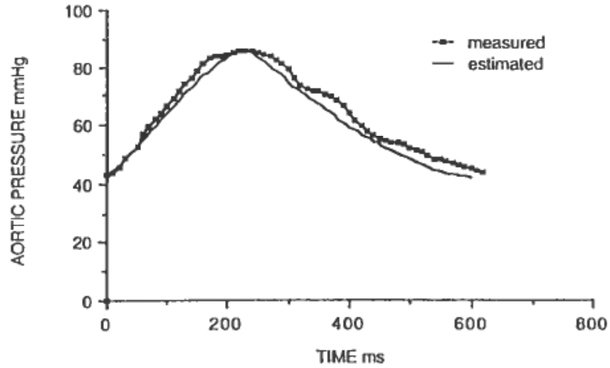

Figure 6. Measured and estimated aortic pressures using the diastolic model of the aorta without inertance *L*.

obtained from the electromagnetic flow measurement. The results are very good with a correlation coefficient of 0·95 over a range of Q_r from 2·1 to 17·8 mL.

The same estimation scheme was then repeated using A_s obtained from 2-D ultrasonic imaging of the aortic orifice, and using the systolic time velocity integral to estimate Q_s, as described in Skjaerpe *et al.* (1985). This was done in just three samples and the results of these measurements are shown as crosses in Fig. 4.

Figure 5 shows the regurgitant orifice obtained by parameter estimation versus the orifice obtained by dividing the regurgitant volume as obtained from the electromagnetic measurements in the ascending aorta, and dividing it by the diastolic time velocity integral DI. Again we see a good correlation between these two estimates ($r=0{\cdot}95$).

Figure 6 shows the estimated aortic pressures in a typical case together with the pressures measured with catheters just to illustrate the similarity. We have used the diastolic model without the inertia both in systole and diastole. The estimated systolic aortic pressure rises slower than the measured pressure because we have neglected the inertia, while in diastole a good resemblance between the estimated and the measured pressures are found. The similarity between the estimated and the measured pressure waveforms indicates that the approximation that was done to obtain the models in Figs. 1 and 2 are acceptable.

6. Discussion

These initial studies in open chested pigs clearly indicates that the simplified models set up for aortic flow can be used for estimating the regurgitant orifice and volume based on ultrasonic Doppler measurements of blood velocities through the aortic valve. To evaluate its full validity, the method has still to be tested in the clinical situation, which has now been started.

The systolic and diastolic model for aortic flow has been discussed elsewhere on numerous occasions. It contains basically four parameters: L, C, R, and A. Figure 6 shows that to obtain the temporal variation of the pressure in the aorta during systole, we cannot neglect the inertia, but by integrating over systole we have been able to eliminate L in (12). In diastole we can neglect L due to the low acceleration of the flow. Using the relations between R, C, and A in (13) and (19) we are left with only one parameter, the regurgitant orifice A, to be estimated from the regurgitant jet velocity.

To do this parameter reduction we have made use of approximated mean systolic and cyclic pressures in (20) and (21). These formulas are approximations, but based on the good correlations in the pig models over a range of regurgitant fractions up to 0·5 the approximations are acceptable.

The conclusion of the study of the pigs is clearly that the method is good when accurate measurements are done. The clinical value of the method has still to be proven, but the prospects are good. It should be pointed out that the regurgitant jet often has directions relative to the beam which makes angle corrections necessary. It is then an advantage to use colour flow imaging of the jet to determine its inclination relative to the CW ultrasonic beam.

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