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Optimal continuous-path control for manipulators with redundant degrees of freedom

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A control system for macro–micro manipulators is presented. A position transformation from the end-effector reference to joint coordinates is found using kinematic optimization. Decoupling and optimal control is used to coordinate the motion of the macro and micro part. The redundant manipulator will then have the speed of the micro manipulator and the large workspace of the macro manipulator. When optimal control is used, the redundant manipulator may be even faster than the micro manipulator provided that a suitable performance index is used. The performance of the manipulator is optimized over the whole reference, and this will give better results than the purely kinematic instantaneous optimization which is the dominating technique in research literature.

1. Introduction

Manipulators with redundant degrees of freedom offer several advantages such as singularity and obstacle avoidance and improved dynamic characteristics. However, improved performance is only obtained if a good control strategy is used.

Most techniques for redundancy resolution are based on optimization. Velocity transformation methods where generalized inverses of the manipulator Jacobian are used, have been employed for the instantaneous minimization of joint velocities (Whitney 1972), kinetic energy, and torques (Hollerbach and Suh 1987). As joint positions are not controlled when these techniques are used, the manipulator may approach singular positions or lose degrees of freedom when joints reach their limit. Gradient projection methods have been used to maximize manipulability (Yoshikawa 1985) and available joint range (Liegois 1977). This results in a hill-climbing search method which is partially mechanically implemented.

A problem with the velocity transformation methods is that they are generally not cyclic. Baker and Wampler (1988) proved that if a velocity transformation is cyclic, the transformation has an equivalent inverse kinematic function which is a position transformation.

Egeland (1987a) and Ahmad and Luo (1988) used position transformations for redundancy resolution by optimizing an objective functional which made the manipulator avoid singular positions and the loss of degrees of freedom. Manipulability or available joint range may be instantaneously optimized in this way without

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having to use the relatively slowly converging hill-climbing method. Chang (1987) discussed position transformations for a general objective functional.

Although kinematic optimization is widely used to control redundant manipulators, little attention has been given to optimal control of this type of manipulator. Kinematic redundancy resolution is the main problem in singularity avoidance and to some extent in obstacle avoidance. However, the use of redundant degrees of freedom to improve the dynamic characteristics of the manipulator (Egeland 1987a; Khatib 1988; Salisbury and Abramowitz 1985) is primarily a control problem.

A macro–micro manipulator is a redundant manipulator with the large workspace of the macro manipulator which is the positioning part, and in addition the accuracy and speed of the micro manipulator mounted on the larger macro part. The dynamic characteristics of the total manipulator may in fact be better than those of the micro manipulator (Khatib 1988) provided that an adequate control strategy is used.

Hsu, Hauser and Sastry (1988) and Khatib (1988) used a generalized inverse of the Jacobian to decouple the end-effector motion, while gradient projection was used to control the nullspace motion.

In our work, we used a position transformation for redundancy resolution. The manipulator was then controlled using optimal control theory. In the simplest implementation, the end-effector and nullspace motion was decoupled, while in the more advanced implementation the actuator torques were weighted against control deviations in the performance index. This resulted in a fine coordination of the macro and micro motions, which was demonstrated in simulation experiments.

2. Statement of the problem

An n-link manipulator is described by the model

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} = \boldsymbol{n}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{\tau} \tag{1}$$

where $\boldsymbol{q}$ is the vector of the joint coordinates, $M(\boldsymbol{q})$ is the positive definite inertia matrix, $\boldsymbol{n}(\boldsymbol{q}, \dot{\boldsymbol{q}})$ is a vector consisting of Coriolis, centrifugal gravity and friction terms, and $\boldsymbol{\tau}$ is the vector of input generalized forces.

The end-effector coordinates $\boldsymbol{p}$ are given by

$$\boldsymbol{p} = \boldsymbol{h}(\boldsymbol{q}) \tag{2}$$

where the first three components of $\boldsymbol{p}$ are the Cartesian positions of hand, and the last three are the appropriate Euler angles. The end-effector velocity is given by

$$\dot{\boldsymbol{p}} = J(\boldsymbol{q})\dot{\boldsymbol{q}} \tag{3}$$

where $J(\boldsymbol{q})$ is the manipulator Jacobian. The system may be decoupled in end-effector coordinates (Freund 1982; Luh, Walker and Paul 1980) using

$$\boldsymbol{\tau} = M(\boldsymbol{q})J^{-1}(\boldsymbol{q})[\boldsymbol{u} - \dot{J}(\boldsymbol{q})\dot{\boldsymbol{q}}] - \boldsymbol{n}(\boldsymbol{q}, \dot{\boldsymbol{q}}) \tag{4}$$

which results in the equivalent model

$$\ddot{\boldsymbol{p}} = \boldsymbol{u} \tag{5}$$

The problem is to track the end-effector reference $\boldsymbol{p}_0(t)$ using one of the models (1) or (5).

3. Redundant degrees of freedom—kinematic optimization

Redundancy resolution by kinematic optimization may be used to obtain a position or velocity reference in joint space which avoids singularities, joint limits and obstacles.

This can be done by solving the optimization problem

$$\max_{q_0} L(q_0) \tag{6}$$

when

$$p_0 = h(q_0)$$

Here, L is the performance functional, p_0 is the end-effector reference, q_0 is the joint reference and

$$\dim (p) = m < \dim (q) = n$$

For singularity avoidance, the performance functional should be the manipulability (Yoshikawa, 1985)

$$L_M = \sqrt{\det [J(q)J^T(q)]} \tag{7}$$

The available joint range may be optimized using (Liegois 1977)

$$L_{JR} = (q - q_c)^T Q_{JR}(q - q_c) \tag{8}$$

where q_c is the centre of the range of travel for q and Q_{JR} is a weighting matrix.

Obstacle avoidance is achieved using

$$L_{obst} = L_{obst}(q, q_{obst}) \tag{9}$$

A combination of L_M, L_{JR} and L_{obst} may be used in (6), alternatively constrained optimization may be used to avoid singularities, joint limits or obstacles. The optimization problem (6) may be solved using e.g. the gradient projection method.

Other performance functionals which have been used are joint velocity (Whitney 1972).

$$L = \dot{q}^T \dot{q} \tag{10}$$

kinetic energy

$$L = \tfrac{1}{2}\dot{q}^T M(q)\dot{q} \tag{11}$$

and torque (Hollerbach and Suh 1987)

$$L = \tau^T \tau \tag{12}$$

The last objective functionals (10)–(12) are associated with the dynamic behaviour of the manipulator. However, this is a control problem which cannot be adequately solved by kinematic methods (Lunde, Egeland and Balchen, 1986) but rather by decoupling or optimal control. The main weakness of kinematic optimization in this case is that instantaneous optimization is used, whereas in optimal control, the time integral of an objective functional is optimized.

4. Cartesian decoupling in macro–micro manipulator control

Our control concept for a small, fast manipulator (micro manipulator) on a large positioning part (macro manipulator) was inspired by the motion of the highly redundant human arm. The fingers and wrist move with small, fast motions, while the elbow, shoulder and body provides a slow, gross motion. This distribution of fast and slow motion can be achieved for macro–micro manipulators by cartesian decoupling.

The macro manipulator, which is connected to the ground, has n_M degrees of freedom and the joint coordinates are denoted by $\boldsymbol{q}_M$. The micro-manipulator, which is mounted on the end of the macro part, has n_μ degrees of freedom and the joint coordinates are denoted $\boldsymbol{q}_\mu$. The resulting macro–micro manipulator has $n = n_M + n_\mu$ degrees of freedom and the generalized joint coordinates are $\boldsymbol{q} = [\boldsymbol{q}_M^T, \boldsymbol{q}_\mu^T]^T$. If m represents the number of end-effector degrees of freedom, n_M and n_μ are assumed to obey

$$n_M \geqslant 1 \quad \text{and} \quad n_\mu = m$$

In the decoupling of non-redundant manipulators, the number of end-effector coordinates equals the number of joint coordinates. This can be achieved for macro–micro manipulators by simply augmenting the end-effector coordinate vector $\boldsymbol{p}$ with the end-point coordinate vector $\boldsymbol{p}_M$ of the macro manipulator (Egeland 1987a). The dimension of $\boldsymbol{p}_M$ is

$$\dim(\boldsymbol{p}_M) = \dim(\boldsymbol{q}_M) = n_M$$

It is here assumed that the macro part is non-redundant with respect to its end-point. The extension to redundant macro manipulators can be done by including the coordinates of additional intermediate points in $\boldsymbol{p}_M$ so that the components of $\boldsymbol{p}_M$ constitute a set of generalized coordinate for the macro part.

The augmented end-effector coordinate vector is written

$$\boldsymbol{p}_A = \begin{bmatrix} \boldsymbol{p}_M \\ \boldsymbol{p} \end{bmatrix} \tag{13}$$

where

$$\dim(\boldsymbol{p}_A) = n$$

as it is assumed that $m = n_\mu$.

The end-effector reference $\boldsymbol{p}_0$ is given, while a reference $\boldsymbol{p}_{M0}$ to the macro manipulator may be found by a kinematic optimization as in (6). The augmented end-effector reference is then

$$\boldsymbol{p}_{A0} = [\boldsymbol{p}_{M0}^{\mathrm{T}} \boldsymbol{p}_0^{\mathrm{T}}]^{\mathrm{T}}.$$

We define the augmented Jacobian by

$$J_A(\boldsymbol{q}) = \frac{\partial \boldsymbol{p}_A}{\partial \boldsymbol{q}} \tag{14}$$

The augmented Jacobian is square and non-singular almost everywhere.

Decoupling may now be applied in the augmented end-effector coordinates (Egeland 1987a) analogously with the resolved acceleration control scheme using

$$\tau = M(\boldsymbol{q})J_A^{-1}(\boldsymbol{q})[\boldsymbol{u}_A - \dot{J}_A(\boldsymbol{q})\dot{q}] - \boldsymbol{n}(\boldsymbol{q}, \dot{\boldsymbol{q}}) \tag{15}$$

which gives n decoupled double integrators:

$$\ddot{\boldsymbol{p}}_A = \boldsymbol{u}_A \tag{16}$$

or alternatively

$$\ddot{p}_M = u_M \tag{17}$$

$$\ddot{p} = u \tag{18}$$

where $u_A = [u_M^T, u^T]^T$.

The end-effector motion, described by p, is then decoupled from the internal or nullspace motion described by p_M. This makes it possible to specify a high bandwidth for the end-effector motion, and a lower bandwidth for the internal motion of the macro manipulator. The end-effector will then have the bandwidth and speed of the fast micro manipulator, while the low bandwidth and possibly large control deviation of the macro manipulator are compensated for by the fast micro part. At the same time a large workspace is obtained by controlling p_M.

5. Optimal control of macro–micro manipulators

5.1. *Introduction*

The use of decoupling to control a macro–micro manipulator gives good results, but it is our conjecture that the manipulator will not have better dynamic characteristics than the micro manipulator with this control scheme. The reason for this is that the motion of the macro and micro part are decoupled, so it is not possible to obtain dynamic characteristics that are better than those of the micro part by coordinating macro and micro motion. However, this can be done using optimal control theory provided that a suitable performance index is used. We have found that inclusion of the input generalized forces τ instead of the acceleration u in the performance index may give the desired coordination of macro and micro motion.

5.2. *Review of the linear quadratic tracking problem*

A reference $y_0(t)$, $t_0 < t < t_f$, is given to the linear system

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) \tag{19}$$

$$y(t) = D(t)x(t) \tag{20}$$

Accurate tracking without excessive control energy can be achieved using a linear quadratic performance index:

$$J(u) = \tfrac{1}{2}\Delta y^T(t_f)S\Delta y(t_f) + \frac{1}{2}\int_{t_0}^{t_f} [\Delta y^T(t)Q(t)\Delta y(t) + u^T(t)P(t)u(t)]\, dt \tag{21}$$

where $\Delta y = y - y_0$ is the control deviation, S and Q positive semi-definite matrices and P is a positive definite matrix.

The optimal control is then (Athans and Falb, 1966)

$$u(t) = G(t)x(t) - P^{-1}(t)B^T(t)h(t) \tag{22}$$

where

$$G(t) = -P^{-1}(t)B^T(t)R(t) \tag{23}$$

$\boldsymbol{R}$ is the positive definite solution of the Riccati equation

$$\dot{R}(t) = -R(t)A(t) - A^{\mathrm{T}}(t)R(t) + R(t)B(t)P^{-1}(t)B^{\mathrm{T}}(t)T(t) - D^{\mathrm{T}}(t)Q(t)D(t) \tag{24}$$

with boundary condition

$$R(t_f) = D^{\mathrm{T}}(t_f)SD(t_f) \tag{25}$$

$\boldsymbol{h}$ is found from

$$\dot{\boldsymbol{h}}(t) = -[A(t) + B(t)G(t)]^{\mathrm{T}}\boldsymbol{h}(t) + D^{\mathrm{T}}(t)Q(t)\boldsymbol{y}_0(t) \tag{26}$$

with boundary condition

$$\boldsymbol{h}(t_f) = -D^{\mathrm{T}}(t_f)S_{y_0}(t_f) \tag{27}$$

The optimal trajectory is the solution of the linear differential equation

$$\dot{\boldsymbol{x}}(t) = [A(t) + B(t)G(t)]\boldsymbol{x}(t) - B(t)P^{-1}(t)B^{\mathrm{T}}(t)\boldsymbol{h}(t) \tag{28}$$

System (26) is the adjoint of (28). $\boldsymbol{h}$ is found by solving (26) backward in time from t_f. This can be done off-line when $\boldsymbol{y}_0(t)$, $t_0 \leqslant t \leqslant t$, is known in advance.

It is seen from (22) that the controller uses information from future values of the reference.

5.3. *Macro–micro manipulators*

A controller is easily found for the n decoupled double integrators (17) and (18) with the performance index

$$J_1(\boldsymbol{u}_A) = \tfrac{1}{2}\Delta\boldsymbol{p}^{\mathrm{T}}(t_f)S\Delta\boldsymbol{p}(t_f) + \tfrac{1}{2}\Delta\boldsymbol{p}_M^{\mathrm{T}}(t_f)S_M\Delta\boldsymbol{p}_M(t_f) + \frac{1}{2}\int_{t_0}^{t_f}(\Delta\boldsymbol{p}^{\mathrm{T}}Q\Delta\boldsymbol{p} + \Delta\boldsymbol{p}_M^{\mathrm{T}}Q_M\Delta\boldsymbol{p}_M + \boldsymbol{u}^{\mathrm{T}}P\boldsymbol{u} + \boldsymbol{u}_M^{\mathrm{T}}P_M\boldsymbol{u}_M)\,dt \tag{29}$$

where $S = \mathrm{diag}\ \{S_{ii}\}$, $S_M = \mathrm{diag}\ \{S_{ii}^{\mathrm{M}}\}$, $Q = \mathrm{diag}\ \{Q_{ii}\}$, $Q_M = \mathrm{diag}\ \{Q_{ii}^{\mathrm{M}}\}$, $P = \mathrm{diag}\ \{P_{ii}\}$ and $P_M = \mathrm{diag}\ \{P_{ii}^{\mathrm{M}}\}$ are constant matrices. $\Delta\boldsymbol{p} = \boldsymbol{p} - \boldsymbol{p}_0$, $\Delta\boldsymbol{p}_M = \boldsymbol{p}_M - \boldsymbol{p}_{M0}$ and $\boldsymbol{p}_0$ and $\boldsymbol{p}_{M0}$ are the references.

If acceleration feedforward is used, (17) and (18) becomes

$$\Delta\ddot{\boldsymbol{p}}_M = \boldsymbol{u}_M \tag{30}$$

$$\Delta\ddot{\boldsymbol{p}} = \boldsymbol{u} \tag{31}$$

and the optimal control is

$$\boldsymbol{u}_M = G_{M1}\Delta\boldsymbol{p}_M + G_{M2}\Delta\dot{\boldsymbol{p}}_M \tag{32}$$

$$\boldsymbol{u} = G_1\Delta\boldsymbol{p} + G_2\Delta\dot{\boldsymbol{p}} \tag{33}$$

where G_{M1}, G_{M2}, G_1 and G_2 are diagonal matrices.

Without acceleration feedforward, the optimal control is (Egeland and Lunde, 1988)

$$\boldsymbol{u}_M = G_{M1}\boldsymbol{p}_M + G_{M2}\dot{\boldsymbol{p}}_M - P_M^{-1}\boldsymbol{h}_{M2} \tag{34}$$

$$\boldsymbol{u} = G_1\boldsymbol{p} + G_2\dot{\boldsymbol{p}} - P^{-1}\boldsymbol{h}_2 \tag{35}$$

where $\boldsymbol{h}_{M2}$ and $\boldsymbol{h}_2$ gives feedforward from future references, and are found from (26) and (27).

As the model is decoupled and the weighting matrices are diagonal, the gain matrices will be diagonal. This means that the closed loop system becomes decoupled.

A more advanced controller which gives the desired coordination of motion can be obtained by using the performance index

$$J_2(\tau) = \frac{1}{2}\int_{t_0}^{t_f} (\Delta \boldsymbol{p}^{\mathrm{T}} Q \Delta \boldsymbol{p} + \Delta \boldsymbol{p}_M^{\mathrm{T}} Q_M \Delta \boldsymbol{p}_M + \tau^{\mathrm{T}} P \tau)\, dt \tag{36}$$

where τ is used instead of $\boldsymbol{u}$ and $\boldsymbol{u}_M$. Q, Q_M and P are constant diagonal matrices, but the weights are different from (29).

The optimal control problem with performance index (36) and model (1) must be solved numerically. However, the problem may be approximated with a linear quadratic problem which is easier to solve when the approximation

$$\tau = M[\boldsymbol{q}_0(t)] J_A^{-1}[\boldsymbol{q}_0(t)] \boldsymbol{u}_A \tag{37}$$

is used where $\boldsymbol{q}_0$ is the joint reference corresponding to the augmented end-effector reference $\boldsymbol{p}_{A0} = [\boldsymbol{p}_{M0}^{\mathrm{T}} \boldsymbol{p}_0^{\mathrm{T}}]^{\mathrm{T}}$. This will be a good approximation provided that the reference is accurately tracked. The non-linear compensation terms have little significance on system stability (Egeland 1986; Spong and Vidyasagar, 1987) and are therefore omitted.

The performance index (36) may now be written

$$J_2(\boldsymbol{u}_A) = \frac{1}{2}\int_{t_0}^{t_f} (\Delta \boldsymbol{p}^{\mathrm{T}} Q \Delta \boldsymbol{p}^{\mathrm{T}} + \Delta \boldsymbol{p}_M^{\mathrm{T}} Q_M \Delta \boldsymbol{p}_M + \boldsymbol{u}_A^{\mathrm{T}} \tilde{P}_A u_A)\, dt \tag{38}$$

where

$$\tilde{P}_A(t) = J_A^{-\mathrm{T}}[\boldsymbol{q}_0(t)] M^{\mathrm{T}}[\boldsymbol{q}_0(t)] P M[\boldsymbol{q}_0(t)] J_A^{-1}[\boldsymbol{q}_0(t)] \tag{39}$$

The model (17) and (18) with the performance index (38) is an LQ problem with optimal control

$$\boldsymbol{u}_A(t) = G_{A1}(t)\boldsymbol{p}_A(t) + G_{A2}(t)\dot{\boldsymbol{p}}_A(t) - \tilde{P}_A(t)^{-1}\boldsymbol{h}_{A2}(t) \tag{40}$$

where $\boldsymbol{h}_{A2}$ is found from (26) and (27). The feedback matrices G_{A1} and G_{A2} are not diagonal and the element of $\boldsymbol{h}_{A2}$ are coupled. This gives the desired coordination of motion.

The control (40) may be looked upon as a sub-optimal solution for the performance index (36) and the model (1). Alternatively, the use of (36) and (37) may be regarded as a systematic way of finding the weighting matrix $\tilde{P}(t)$ in the LQ-problem (17), (18) and (38).

The method requires future references to be known. This is normally the case for industrial robots. However, when sensory feedback from e.g. vision is used, the method has to be modified.

6. Application to an eight-link industrial robot

Decoupling in macro–micro manipulator control has been applied to the Trallfa TRACS spray-painting robot (Fig. 1) in experiments and the results are given in

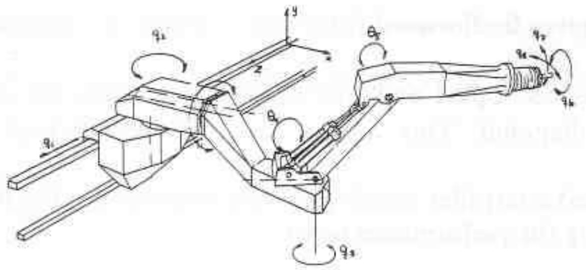

Figure 1. The Trallfa TRACS spray-painting robot.

(Egeland 1987b). It was clearly demonstrated that the total manipulator had as good dynamic characteristics as the fast outer manipulator.

The combination of off-line and real-time optimal control as proposed above was applied to the same manipulator in a simulation study, and the results are presented here.

The TRACS manipulator consists of a six-link non-redundant manipulator mounted on a positioning part which consists of a waggon, and a rotational joint on the waggon. As the joints in the macro manipulator move large inertias, they are not as fast as the outer micro manipulator due to actuator saturation and bandwidth limitations.

The translational displacement of the waggon is denoted by q_1, the joint angle in the positioning part by q_2, and the angles of the outer non-redundant manipulator by q_3 to q_8. The end-effector coordinate vector is

$$p = [x, y, z, \phi, \theta, \psi]^{\mathrm{T}} \tag{41}$$

where x, y and z are the end effector coordinates, and ϕ, θ and ψ are the respective roll, pitch and yaw angles of the end effector.

An end-effector reference p_0 was given. The generalized coordinates of the positioning part were selected as

$$p_M = [z_1, z_2]^{\mathrm{T}} \tag{42}$$

where z_1 is the z coordinate of the base of the outer manipulator and $z_2 = q_1$ is the position of the waggon (Fig. 2).

The micro manipulator has no singularities within its workspace, so kinetic optimization was only used to avoid limits in the simulations.

Instead of optimizing the available joint space for the micro manipulator numerically, an analytic approximation was used. The kinematic redundancy was

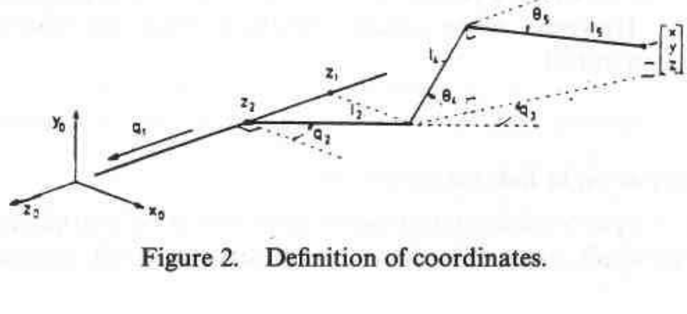

Figure 2. Definition of coordinates.

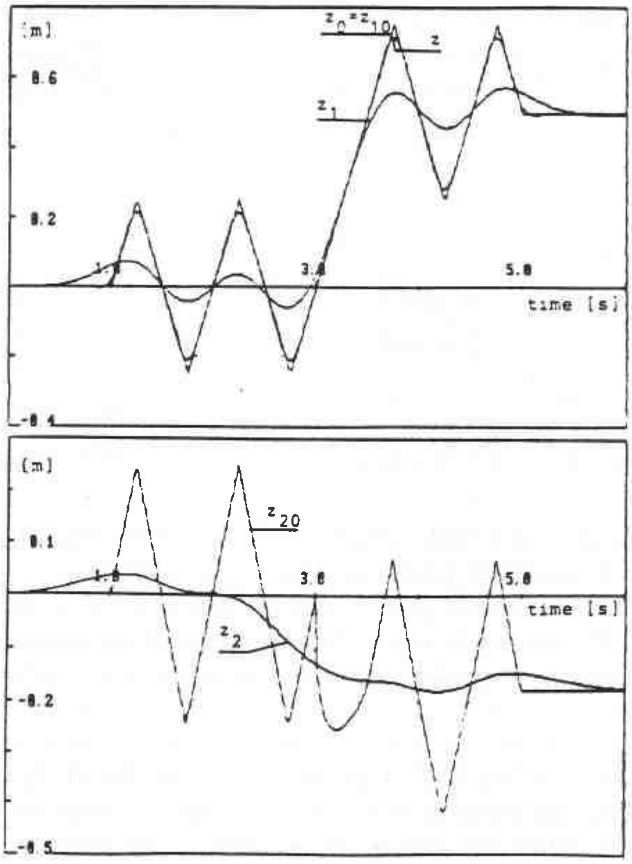

Figure 3. End-effector (z_0), end-effector response (z), macro part endpoint (z_1) and waggon (z_2) when decoupling was used in simulation.

resolved by simply using

$$l_4 \cos \theta_4 + l_5 \cos \theta_5 = l_5$$

$$q_8 = 0$$

where l_4 and l_5 are the lengths of link 4 and 5 (Fig. 2).

With these two additional equations, a reference

$$\boldsymbol{p}_{M0} = [z_{10}, z_{20}]^{\mathrm{T}}$$

to the macro part could be found.

In the first simulation the performance index was

$$J_1(\boldsymbol{u}_A) = \int_0^{\infty} (\Delta \boldsymbol{p}^{\mathrm{T}} Q \Delta \boldsymbol{p} + \Delta \boldsymbol{p}_M^{\mathrm{T}} Q_M \Delta \boldsymbol{p}_M + \boldsymbol{u}^{\mathrm{T}} P \boldsymbol{u} + \boldsymbol{u}_M^{\mathrm{T}} P_M \boldsymbol{u}_M)\, dt$$

where $Q = 1{\cdot}6 \times 10^5 I_3$, $Q_M = \mathrm{diag}\,\{400, 50\}$, $P = I_3$ and $P_M = I_2$.

The optimal control was given by (34) and (35). The wrist angles were controlled with proportional feedback.

To achieve fast and accurate tracking of the end-effector reference $\boldsymbol{p}_0$, the weighting factors in Q were high, while the weighting factors in Q_M associated with the nullspace motion were moderate. As a result of this, the micro manipulator tracked

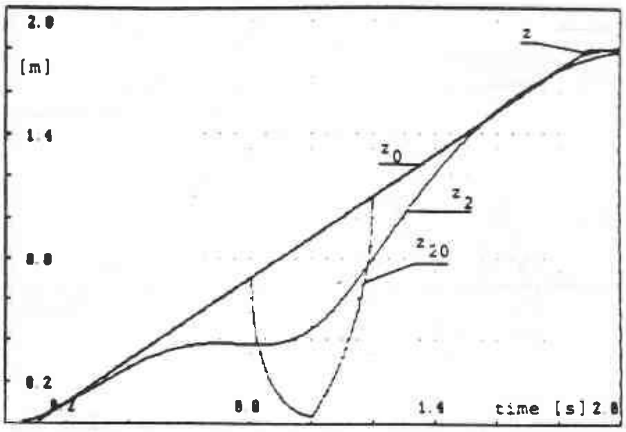

Figure 4. Tracking of saw-tooth in x. Decoupling was used.

the reference accurately and with a high bandwidth, while the macro part did the slow, gross positioning so that a large workspace was obtained.

A saw-toothed reference was given in the z direction which is the direction of the rail. After 2·75 s the saw-tooth was shifted 0·5 m, and the same time there was a ramp in the x direction. The velocity in the z direction was 1 m/s, and the y reference was constant.

The optimal control law (34) and (35) gave accurate tracking of the end-effector reference with a band-width that exceeded the bandwidth of the positioning part (Fig. 3). The optimal feedforward made the slow macro manipulator start to move to the new position before the shift in the end-effector reference. It was clearly seen that the micro manipulator did the high frequency motion, while the macro part did the low frequency gross positioning.

A ramp of 1 m/s in the z direction and a single saw-tooth of $\pm 0{\cdot}3$ m/s in the x direction was then tracked. The macro part moved to keep the micro part close to the centre of its workspace as is seen from Figs. 4 and 5.

In the second simulation the performance index was

$$J_2(\tau) = \int_{t_0}^{t_f} (\Delta p^{\mathrm{T}} Q \Delta p + \Delta p_M^{\mathrm{T}} Q_M \Delta p_M + \tau^{\mathrm{T}} P \tau)\, dt$$

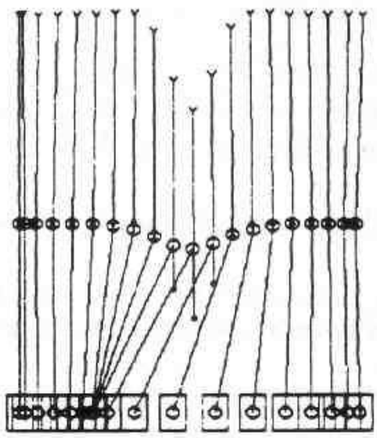

Figure 5. Top view of the manipulator corresponding to simulation in Fig. 4.

where $Q = 0{\cdot}01\ I_5$. $Q_M = 1 \cdot 10^3\ I_3$ and $P = 0{\cdot}01\ I_5$. The optimal control was given by (40), and the wrist angles were again controlled by proportional controllers. The elements in the weighting matrices were computed from

$$Q_{ii} = (\Delta p_i^{\max})^{-2}, \quad Q_{ii}^M = (\Delta p_{Mi}^{\max})^{-2}$$

and

$$P_{ii} = (\tau_i^{\max})^{-2},$$

where $\Delta p_i^{\max}$ and $\Delta p_{Mi}^{\max}$ are the largest acceptable position deviations and $\tau_i^{\max}$ the maximum allowable torque in joint i. The weights Q_{ii} were high and Q_{ii} moderate, as large deviations in the macro manipulator are tolerated as long as the joint limits are not reached.

The saw-tooth reference was tracked once again. The response of the waggon (z_2) and the end effector (z) were similar to the first simulation where acceleration was weighted (Fig. 6). However, the motion of the intermediate point (z_1) was clearly coordinated with the motion of the micro manipulator. When the micro part accelerated in the z direction, the inertia coupling made the intermediate point accelerate the opposite way. This was not compensated for in this simulation as the square of the input generalized forces was minimized in the performance index.

Moreover, this control scheme took advantage of the inertia coupling. The macro part was accelerated in the z direction before an acceleration in the same

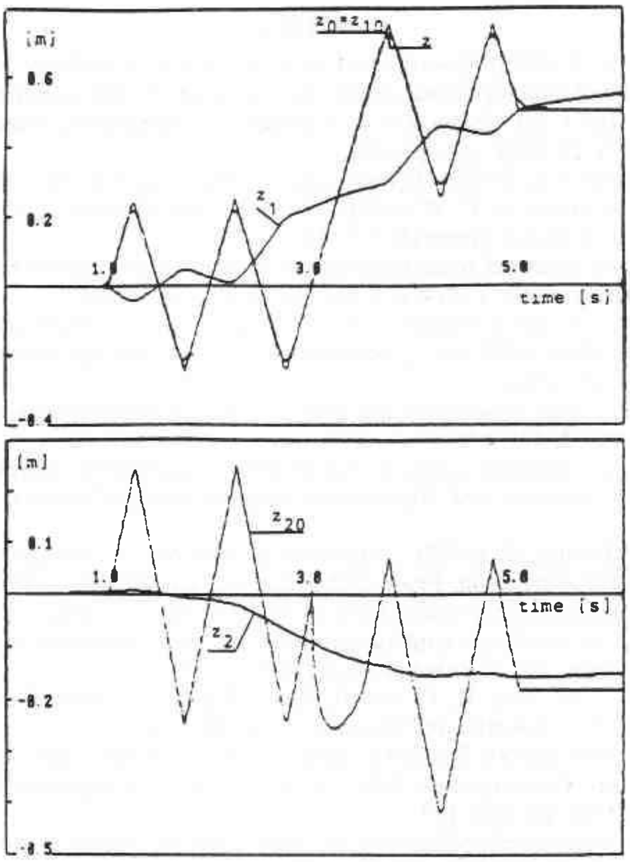

Figure 6. Simulation with τ in the performance index. The motion of macro part endpoint (z_1) is coordinated with the end-effector motion (z).

direction was required in the micro part. When the micro part started to accelerate, the macro transferred its kinetic energy to the micro by accelerating the opposite way. This is a motion which is similar to that of an athlete throwing a javelin where the kinetic energy of the body (the macro part) is transferred to the arm (the micro part) when the javelin is thrown.

7. Conclusion

A control system for the control of macro–micro manipulators has been presented. The kinematic redundancy was resolved by a position transformation of the end-effector reference and optimal control was used to track the reference.

When decoupling was used in simulations, the redundant manipulator was as fast as the micro manipulator. When the input generalized forces were included in the performance index, the redundant manipulator was faster than the micro manipulator.

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