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Cartesian control of a spray-painting robot with redundant degrees of freedom

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A controller for redundant manipulators with a small, fast manipulator mounted on a positioning part has been developed. The controller distributes the fast motion to the small, fast manipulator and the slow, gross motion to the positioning part. A position reference is generated on-line to the positioning part to avoid singularities and the loss of degrees of freedom. This reference is selected according to an *ad hoc* procedure which makes the small, fast manipulator work around the centre of its working range. In the control system, the task space position vector is augmented with the generalized coordinates of the positioning part. The resulting augmented task space vector contains a set of generalized coordinates for the manipulator. Feedback linearization and decoupling are applied in the augmented task space to obtain a model consisting of decoupled double integrators. The low and high frequency motion is distributed by controlling the double integrators associated with the end effector with a high bandwidth, while the double integrators associated with the positioning part are controlled with a low bandwidth.

The controller was applied to the Trallfa TRACS spray-painting robot which is an eight-link robot with hydraulic actuators. In this application, the five inner joints were controlled. Feedback linearization and decoupling were successfully implemented using load pressure feedback to compensate for actuator dynamics. The experiments showed that a high bandwidth and a large working range were possible with moderate motor torques, and that the end effector had a higher bandwidth than the positioning part.

1. Introduction

If a small, fast manipulator is mounted on a large positioning part, this results in a manipulator with a high bandwidth and a large working range. However, the high bandwidth is only obtainable if the control system distributes the fast motion to the small, fast manipulator and the slow, gross motion to the positioning part.

The present study was undertaken to develop this type of control system and to implement it on a Trallfa TRACS spray-painting robot. This is an industrial robot with the same mechanical design as that described above. The robot has a fast, six degrees of freedom manipulator mounted on a positioning part which consists of a waggon and a rotational joint. When this robot is controlled using conventional techniques, the bandwidth of the end effector motion is limited by the relatively slow positioning part. With the control system developed in this work, and the end effector has the same bandwidth as the outer fast manipulator.

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When a small manipulator is mounted on a positioning part, the resulting manipulator is said to be redundant or to have redundant degrees of freedom as it has more joints than required to give the end effector a desired position and orientation.

The idea of putting a small, fast manipulator on a large positioning part was inspired by the motion of the highly redundant human arm. The fingers and the wrist move with small, fast motions, while the elbow, the shoulder and the body provides a slow, gross motion. The human hand also uses its redundancy to avoid obstacles in its working area, which is another important application of redundancy in robotics (Khatib 1985).

In many applications, the reference trajectory for the manipulator is specified in terms of task coordinates which are the position and orientation of the end effector in a task-defined coordinate system. The conventional way of controlling a non-redundant manipulator when the reference trajectory is given in the task space, is to transform the task space reference to a reference in joint coordinates. A controller is then designed to track this reference with a PID controller at each joint (Luh 1983), alternatively, this can be done using the computed torque technique (Bejczy 1974).

When the manipulator is redundant, there is no unique transformation from the task space reference to joint space. For the type of manipulators considered in this work, a simple solution to this problem is to select a suitable reference to the positioning part of the manipulator. Under the assumption that the outer manipulator is non-redundant, this will give a unique transformation from task space to joint space. When the joint space reference is given, the manipulator may be controlled as in the non-redundant case. However, if the end effector is to be positioned accurately with a joint space controller, the control deviation must be small in all the joints as a control deviation in the slow positioning part will produce a position error in the end effector. This means that the bandwidth of the end effector is limited by the slow positioning part.

Salisbury and Abramowitz (1985) discussed redundant manipulators with a small, fast manipulator on a positioning part, and investigated a very simple mechanism of this kind. This consisted of a planar two-link mechanism with rotary joints, which was used to position the end of link 2 in one direction. The motion of the mechanism was divided into external motion and internal or null motion in order to obtain a high bandwidth with moderate power consumption. The external motion was controlled with a high bandwidth, while the internal motion which maintained the desirable configuration had a lower bandwidth.

The concept developed by Salisbury and Abramowitz (1985) has been simplified and used in our design work on a control system for redundant manipulators which have a non-redundant manipulator mounted on a positioning part. In the resultant control system, the task space position vector is augmented with the generalized coordinates of the positioning part. Singularities and the loss of degrees of freedom are avoided by specifying a suitable reference to the positioning part. A high bandwidth is obtained by using feedback linearization and decoupling in the augmented task space, which makes it possible to control the end effector with a high bandwidth and the positioning part with a lower bandwidth.

The control system has been applied to the five inner joints of a Trallfa TRACS spray-painting robot both in simulations and experimentally. The three outer joints which are the wrist joints were not controlled. As the robot has hydraulic actuators, we had to use load pressure feedback in order to implement feedback linearization and decoupling. These experiments are described more fully here.

2. Comparison with previous work

Most previous work on redundant manipulators has been on the kinematical problem of selecting an appropriate joint space motion when the task space motion is specified. In the research literature this problem has been solved by using the manipulator Jacobian to transform the task space velocity or acceleration to the joint space. Since the Jacobian matrix of a redundant manipulator is rectangular and not invertible, generalized inverses of the Jacobian have been used (Klein and Huang 1983; Whitney 1972). By using an appropriate generalized inverse, a performance index that is quadratic in joint veloctities is minimized. However, when this technique is used, the manipulator may reach singularities or lose degrees of freedom (Baillieul 1985) as the joint positions are not controlled.

A vector in the null space of the Jacobian has been added to the joint velocity to improve the result that is obtained from using generalized inverses. To avoid singularities and the loss of degrees of freedom, Yoshikawa (1985) introduced the manipulability functional which was to be minimized. This was done by selecting a null vector in the manipulability gradient direction. In this way the manipulator searches for the configuration with the highest manipulability in a hill-climing search method which is partly mechanically implemented. Hollerbach and Suh (1985) selected the null vector to minimize actuator torques, but in their experiments, singularities were not avoided.

In Lunde, Egeland and Balchen (1986), techniques based on the use of generalized inverses were compared through simulations to the control system proposed in this work. It was shown that when generalized inverses were used, large actuator torques were required in the positioning part as most of the end effector motion including high frequency motion resulted from motion in the positioning part. Further, it was shown that the manipulator approached singular points when generalized inverses were used, even if the manipulability was maximized. None of these problems were experienced with the new control system.

It was our experience that with a manipulator mounted on a positioning part, a reference should be given to the positioning part according to an *ad hoc* procedure. This is the obvious solution, and it is the one that is used in industry and space applications when a manipulator is mounted on a vehicle.

Feedback linearization and decoupling is widely used in robotics. This technique is used to transform complicated, non-linear, coupled state space equations of motion into a set of decoupled double integrators which are easy to control.

This technique has been used to obtain double integrators in the joint space (Bejczy 1974) and in this connection it has been called the computed torque technique or inverse dynamics.

In the task space, the technique has been called resolved acceleration control (Luh, Walker and Paul 1980a). Both the computed torque and the resolved acceleration control scheme are easily derived from the equation of motion for manipulators with rigid joints. Freund (1982) and Tarn, Bejczy, Isidori and Chen (1984) derive the same techniques from the more general theory of differential geometry, feedback linearization and decoupling.

Feedback linearization and decoupling has been proposed for the control of manipulators with electrical motors. In this work, feedback linearization and decoupling is used to control a manipulator with hydraulic actuators, and we believe that this is the first successful implementation of this control strategy for a hydraulic manipulator.

3. Review of feedback linearization and decoupling

The equation of motion for a general n-link manipulator can be found from Newton–Euler's equation (Luh, Walker and Paul 1980b), this is written

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} = \boldsymbol{n}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{\tau} \quad (1)$$

where $\boldsymbol{q}$ is the vector of joint coordinates, $M(\boldsymbol{q})$ is the inertia matrix, $\boldsymbol{n}(\boldsymbol{q}, \dot{\boldsymbol{q}})$ is a vector consisting of friction, gravity, Coriolis and centrifugal terms, and $\boldsymbol{\tau}$ is the vector of input generalized forces, which is considered to be the control vector.

The task space position vector is denoted by $\boldsymbol{p}$, and is given by

$$\boldsymbol{p} = h(\boldsymbol{q}) \quad (2)$$

The relation between the velocity $\dot{\boldsymbol{p}}$ in the task space and the velocity $\dot{\boldsymbol{q}}$ in the joint space is given by

$$\dot{\boldsymbol{p}} = J(\boldsymbol{q})\dot{\boldsymbol{q}} \quad (3)$$

where $J(\boldsymbol{q})$ is the Jacobian matrix defined by $J_{ij} = \partial p_i/\partial q_j$.

By using feedback linearization and decoupling, the model (1) can be transformed into a model consisting of n decoupled double integrators. This is done in the joint space by the computed torque technique (Bejczy, 1974) and in the task space by the resolved acceleration control scheme (Luh, *et al.* 1980a) and in Tarn *et al.* (1984).

In the joint space, the state vector $\boldsymbol{z} = [\boldsymbol{z}_2^T, \boldsymbol{z}_2^T]^T$ is used where $\boldsymbol{z}_1 = \boldsymbol{q}$ and $\boldsymbol{z}_2 = \dot{\boldsymbol{q}}$. The control vector is selected as

$$\boldsymbol{\tau} = M(\boldsymbol{q})\boldsymbol{u} - \boldsymbol{n}(\boldsymbol{q}, \dot{\boldsymbol{q}}). \quad (4)$$

The resulting state space model is

$$\dot{\boldsymbol{z}}_1 = \boldsymbol{z}_2 \quad (5a)$$

$$\dot{\boldsymbol{z}}_2 = \boldsymbol{u} \quad (5b)$$

which is n decoupled double integrators. The transormed control vector $\boldsymbol{u}$ can now be determined for this model (5) using linear control theory, and the input generalized forces $\boldsymbol{\tau}$ are given by (4).

The state vector $\tilde{\boldsymbol{z}} = [\tilde{\boldsymbol{z}}_1^T, \tilde{\boldsymbol{z}}_2^T]^T$ is used for the task space, where $\tilde{\boldsymbol{z}}_1 = \boldsymbol{p}$ and $\tilde{\boldsymbol{z}}_2 = \dot{\boldsymbol{p}}$. If the Jacobian matrix $J(\boldsymbol{q})$ has full rank, the control vector can be selected as

$$\boldsymbol{\tau} = M(\boldsymbol{q})J^{-1}(\boldsymbol{q})[\boldsymbol{u} - \dot{J}(\boldsymbol{q})\dot{\boldsymbol{q}}] - \boldsymbol{n}(\boldsymbol{q}, \dot{\boldsymbol{q}}). \quad (6)$$

Differentiation of (3) gives

$$\ddot{\boldsymbol{p}} = \dot{J}(\boldsymbol{q})\dot{\boldsymbol{q}} + J(\boldsymbol{q})\ddot{\boldsymbol{q}} \quad (7)$$

The state space model

$$\dot{\tilde{\boldsymbol{z}}}_1 = \tilde{\boldsymbol{z}}_2 \quad (8a)$$

$$\dot{\tilde{\boldsymbol{z}}}_2 = \boldsymbol{u} \quad (8b)$$

is obtained from (1), (6) and (7), which again is n decoupled double integrators.

4. Cartesian control of a fast manipulator on a positioning part

4.1. Kinematics

There is no unique transformation from a task space trajectory to a corresponding joint space trajectory for a redundant manipulator.

The problem of tracking a task space reference trajectory with a redundant manipulator can be divided into two areas, first the kinematical problem of avoiding singularities and the loss of degrees of freedom, and second, the control problem of following the trajectory without excessive input generalized forces.

We must be able to control the configuration of the arm in order to avoid singularities and the loss of degrees of freedom. For a manipulator with a non-redundant manipulator mounted on a positioning part, the configuration is given by the task space position $\boldsymbol{p}$ and the generalized coordinates $\tilde{\boldsymbol{p}}$ of the positioning part.

The augmented task space position vector is defined as

$$\boldsymbol{p}_A = \begin{bmatrix} \boldsymbol{p} \\ \tilde{\boldsymbol{p}} \end{bmatrix}$$

where dim $(\boldsymbol{p}_A) = n$, which is the number of joints in the redundant manipulator. The components of $\boldsymbol{p}_A$ constitute a set of generalized coordinates for the manipulator.

When the task space reference $\boldsymbol{p}_{\text{ref}}$ is given, a reference $\tilde{\boldsymbol{p}}_{\text{ref}}$ to the positioning part can be specified, which will give the manipulator a configuration with good kinematic properties.

The $\tilde{\boldsymbol{p}}_{\text{ref}}$ chosen in this investigation makes the non-redundant outer manipulator move close to the centre of its working range.

When the positioning part of the redundant manipulator is non-redundant, $\tilde{\boldsymbol{p}}$ is chosen as the task space position of the base of the outer non-redundant manipulator. This is illustrated in Fig. 1, which shows a three-link manipulator mounted on a three-link positioning part.

If the positioning part has three degrees of freedom and is used to give the base of the outer manipulator three translational degrees of freedom, $\tilde{\boldsymbol{p}}_{\text{ref}}$ in many cases can be chosen as

$$\tilde{\boldsymbol{p}}_{\text{ref}} = \boldsymbol{d}_{\text{ref}} - \boldsymbol{d}_0. \qquad (9)$$

Here $\boldsymbol{d}_{\text{ref}}$ is the translational part of $\boldsymbol{p}_{\text{ref}}$ and $\boldsymbol{d}_0$ is a vector which is constant relative to the base of the outer non-redundant manipulator. $\boldsymbol{d}_0$ is the translational position

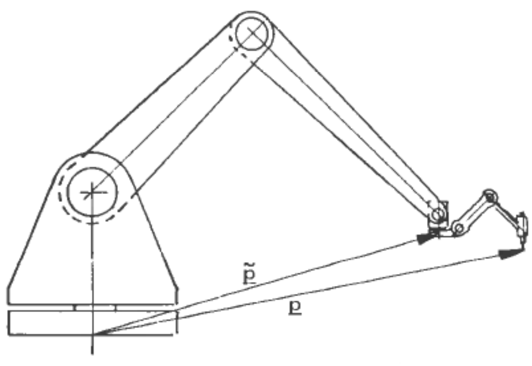

Figure 1. Redundant manipulator.

of the centre of the working range of the outer nonredundant manipulator with respect to its base. If the positioning part has one or two degrees of freedom, $\tilde{p}_{\text{ref}}$ can be chosen as

$$\tilde{p}_{\text{ref}} = d^p_{\text{ref}} - d^p_0 \tag{10}$$

where d^p_{ref} and d^p_0 are the projections of d_{ref} and d_0 into the degrees of freedom of the base of the outer manipulator.

If there are no singularities close to the centre of the working range of the non-redundant manipulator, this kinematical solution will avoid singularities and the loss of degrees of freedom provided that the reference p_{ref} is not close to the limit of the working range of the redundant manipulator.

4.2. Control

If q is the vector of joint coordinates, the augmented task space position p_A is given by

$$p_A = h_A(q). \tag{11}$$

If p_A is specified properly, there will be an inverse transformation

$$q = h_A^{-1}(p_A). \tag{12}$$

This means that if the augmented task space reference $p_{A,\text{ref}}$ is given, a corresponding joint space reference q_{ref} can be found from the inverse transformation (12). A controller can then be developed in the joint space.

However, there are serious drawbacks with this solution for the particular class of redundant manipulators that are investigated here. If the hand is to be positioned accurately, all of the joint coordinates q_i, $i = 1, \ldots, n$, must be close to the corresponding references $q_{i,\text{ref}}$. This means that when the positioning part is large and slow, and the outer non-redundant manipulator is small and fast, the slow positioning part will limit the bandwidth of the system. This method of controlling the redundant manipulator will also require large input generalized forces in the joints. This is because high accelerations are required in the joints of the positioning part, and these joints have to move high inertias.

We therefore developed our controller in the augmented task space. By using feedback linearization, decoupling and linear quadratic optimal control it is then possible to specify that the task space reference p_{ref} is to be followed accurately, whereas less accuracy is required in the tracking of the reference $\tilde{p}_{\text{ref}}$ to the positioning part. In this way we get a control system where the non-redundant outer manipulator takes care of the high frequency part of the reference p_{ref}, while the positioning part takes care of the large, low frequency motions.

4.3. Controller design

The augmented Jacobian matrix is defined by

$$J_A(q) = \frac{\partial p_A}{\partial q}$$

The augmented Jacobian is an $n \times n$ matrix which is non-singular except, of course, in singular points.

In analogy with the task space feedback linearization and decoupling in § 3, the following control vector is used

$$\tau = M(q)J_A^{-1}(q)[u - \dot{J}_A(q)\dot{q}] - n(q, \dot{q}) \tag{13}$$

This gives the state space model

$$\dot{x}_1 = x_2 \tag{14a}$$

$$\dot{x}_2 = u \tag{14b}$$

where $x_1 = p_A$, $x_2 = \dot{p}_A$ and u is the transformed control vector.

A controller for the system (14) can now be designed using linear control theory. If linear quadratic optimal control is used, a suitable performance index is

$$V = \lim_{T \to \infty} \frac{1}{T} \int_0^T (\Delta x^T Q \Delta x + u^T P u) dt \tag{15}$$

where $x = [x_1^T, x_2^T]^T$, $\Delta x = x - x_{\text{ref}}$, x_{ref} is the state reference, $Q = \text{diag}\{q_{11}, \ldots, q_{2n, 2n}\}$ and $P = \text{diag}\{p_{11}, \ldots, p_{nn}\}$.

A high degree of accuracy is desired for tracking the task space reference. Therefore, the weights q_{ii} corresponding to the position deviations in the task space are high. The tracking of the reference for the positioning part need not be very accurate, and the weights q_{ii} which correspond to the position deviations of the positioning part are moderate. The resulting feedback control for the system (14) with performance index given by ((15) is then (Athans and Falb 1966)

$$u_i = g_i \Delta x_i + g_{i+n} \Delta x_{i+n} \tag{16}$$

where $g_i = -\sqrt{(q_{ii}/p_{ii})}$ and $g_{i+n} = -\sqrt{[2\sqrt{(q_{ii}/p_{ii})} + q_{i+n, i+n}/p_{ii}]}$.
If acceleration feedback is included, the control is

$$u_i = \ddot{x}_{i,\text{ref}} + g_i \Delta x_i + g_{i+n} \Delta x_{i+n} \tag{17}$$

For a non-redundant manipulator, the control given by (17) will be as in the resolved acceleration control scheme (Luh *et al.*, 1980a).

In our experiments, we used a controller with a structure as given by linear quadratic control with the performance index (15). However, the feedback gains were selected using pole placement techniques.

5. Application to the Trallfa TRACS spray-painting robot

5.1. Introduction

The Trallfa TRACS spray-painting robot which is shown in Fig. 2 has nine degrees of freedom to position the end effector in six degrees of freedom. Eight of the joints are controlled dynamically, while the joint angle of the ninth joint is either 0° or 180°, depending on the task.

The TRACS manipulator consists of a six-link non-redundant manipulator mounted on a positioning part. The positioning part consists of a translational joint which is a waggon, and a rotational joint mounted on the waggon. The two joints in the positioning part move large inertias, and have a lower bandwidth than the outer non-redundant manipulator.

The TRACS manipulator is designed for the spray-painting of cars. The waggon gives gross positioning along the conveyor. In addition, the rotational joint in the

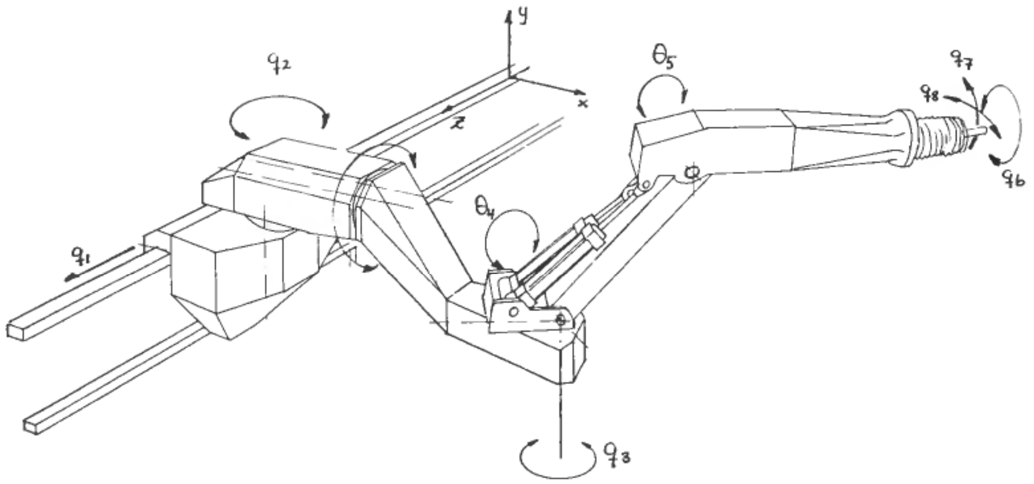

Figure 2. The Trallfa TRACS spray-painting robot.

positioning part also gives a gross position, and can furthermore be used for collision avoidance. When a car is painted, the fast, outer manipulator gives a high bandwidth, while the positioning part gives a large working range.

The main problem in the control of the manipulator is the coupling between the joint motions. When a conventional control strategy is used with one proportional controller at each joint, the inertia coupling between the positioning part and the outer manipulator gives larger control deviations than desired particularly in the rotational joint of the positioning part. Though this joint has an inertia load which is about ten times as high as the inertia load of the first joint in the outer manipulator, the two joints have identical hydraulic actuators. More powerful actuators in the positioning part will probably not give any improvement as this would introduce more kinetic energy into the system and thereby excite elastic motion in the arm. Another problem is the coupling between joints 4 and 5, the piston-driven joints in the outer manipulator.

An early version of the control system presented in § 4 had been developed (Egeland, 1985) when we first started work on the TRACS robot in 1985. The control system seemed to be the solution to the problems encountered in the control of the robot, and we decided to implement the system on the TRACS.

The most interesting problems were associated with the five inner joints, consequently we implemented the controller for these joints. The three outer wrist joints were not controlled. Because of this, the controller algorithm was simplified and the instrumentation became cheaper.

5.2. Dynamic model

The dynamic model of a robotic manipulator contains many terms. As a rule, it is very time consuming to develop an analytical model for a manipulator with more than three joints, and errors may occur in the final model. This problem can be solved by computer-aided modeling of manipulators. The most widely used technique is the recursive Newton–Euler method (Luh *et al.* 1980b). This is an algorithm which computes

$$\tau = M(q)u - n(q, \dot{q}) \tag{18}$$

in the computed torque technique. The dynamic model is not developed when this technique is used. Recently, algorithms which generate analytic models have appeared. In Burdick (1986) a lisp-based program was used.

Because of the special geometry of the TRACS robot, it was possible to develop a dynamic model for the five inner joints manually, which was used instead of the computer-aided modeling.

The translational displacement of the waggon is denoted by q_1, the joint angle of the rotational joint in the positioning part by q_2, and the joint angles of the outer non-redundant manipulator by $q_3, \theta_4, \theta_5, q_6, q_7$ and q_8.

In this application, the five inner joints were controlled.

The vector of joint angles is denoted by

$$\theta = [q_1, q_2, q_3, \theta_4, \theta_5]^T$$

while the generalized coordinates q are

$$q = [q_1, q_2, q_3, q_4, q_5]^T$$

where q_4 and q_5 are the displacements of the hydraulic pistons driving joints 4 and 5. If the inertia of the pistons are neglected, the dynamic model

$$M_\theta(\theta)\ddot{\theta} = n_\theta(\theta, \dot{\theta}) + \sigma \tag{19}$$

can be developed using Newton–Euler's equation. In (19) σ are the generalized forces associated with θ.

By introducing the Jacobian $J_{q\theta} = \partial q/\partial\theta$ we get

$$\dot{q} = J_{q\theta}\dot{\theta} \tag{20}$$

and

$$\sigma = J_{q\theta}^T \tau \tag{21}$$

where τ is the generalized forces from the actuators. By combining (19), (20) and (21) we get the model

$$M(q)\ddot{q} = n(q, \dot{q}) + \tau \tag{22}$$

where

$$M = J_{q\theta}^{-T} M_\theta J_{q\theta}^{-1}$$

and

$$n \cong J_{q\theta}^{-T} n_\theta$$

If the actuators had been DC motors, the model for the manipulators with actuators could have been written in the form

$$\tilde{M}(q)\ddot{q} = \tilde{n}(q, \dot{q}) + u_a \tag{23}$$

where u_a is the control vector to the actuators and $\tilde{M}$ and $\tilde{n}$ are given by e.g. Tarn *et al.* (1984). The model (23) has the same structure as (22), and the control system presented above can therefore be applied to a manipulator with electrical actuators.

Here, hydraulic actuators are used, and one additional state has to be introduced for each actuator (Merritt 1967). We used the load pressure as the additional state variable, and this gives the following model for the manipulator with actuators:

$$\dot{\boldsymbol{p}}_L = C\boldsymbol{p}_L + D_m\dot{\boldsymbol{q}} + B\boldsymbol{u}_v \tag{24}$$

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} = \boldsymbol{n}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + D_t\boldsymbol{p}_L \tag{25}$$

where $\boldsymbol{p}_L$ is the vector of load pressures in the actuators, $\boldsymbol{u}_v$ is the servo valve control vector, and C, D_m, B and D_t are constant diagonal matrices.

It is seen from (25) that if the load pressure $\boldsymbol{p}_L$ can be controlled, a model with the same structure as (22) is obtained. We therefore used load pressure feedback in an internal loop. The control vector $\boldsymbol{u}_v$ was generated by

$$\boldsymbol{u}_v = K_p(\boldsymbol{p}_{L,\,\mathrm{ref}} - \boldsymbol{p}_L) \tag{26}$$

where $\boldsymbol{p}_{L,\,\mathrm{ref}}$ is the load pressure reference, and K_p is a diagonal feedback gain matrix. The feedback gains in (26) should be high to give the internal load pressure feedback loops a higher bandwidth than the total system, and to minimize the influence of the term $D_m\dot{\boldsymbol{q}}$ in (24). The feedback gains are, however, limited by the bandwidth of the electrohydraulic servovalves. If the gains are not high enough, the term $D_m\dot{\boldsymbol{q}}$ may have to be compensated for.

Provided that the load pressure feedback loops are effective, the model

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} = \boldsymbol{n}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + D_t\boldsymbol{p}_{L,\,\mathrm{ref}} \tag{27}$$

is obtained, where D_t is diagonal, and it is then possible to apply the control system presented above.

5.3. Control of redundant degrees of freedom

In an experiment, a position reference was given to the end effector. The reference was specified in terms of the task space position $\boldsymbol{p} = [x, y, z]^T$ which is the position of the end effector in the Cartesian base coordinate system $(x_0\, y_0\, z_0)$. This is a convenient way to specify a reference in spray-painting.

Generalized cordinates for the positioning part had to be selected to apply the control system described in § 4. The configuration of the positioning part may be specified by $\tilde{x}$ and $\tilde{z}$ which are the x_0 and z_0 coordinates of the base of the outer manipulator in the base coordinate system. However, if $\boldsymbol{p}$ is augmented by $\tilde{x}$ and $\tilde{z}$ to $\boldsymbol{p}_A$, the augmented Jacobian $J_A = \partial\boldsymbol{p}_A/\partial\boldsymbol{q}$ will not have full rank and cannot be inverted when $q_2 = 0$. In this experiment, angle q_2 was expected to vary around $q_2 = 0$, which is the value of q_2 when link 2 is normal to the rail of joint 1.

We consequently used the generalized coordinates $\tilde{z}_1$ and $\tilde{z}_2$ where $\tilde{z}_1$ is the z_0 coordinate of the base of the outer manipulator and $\tilde{z}_2$ is the z_0 coordinate of the waggon. This gave

$$\boldsymbol{p}_A = \begin{bmatrix} x \\ y \\ z \\ \tilde{z}_1 \\ \tilde{z}_2 \end{bmatrix} = \begin{bmatrix} C_2 l_2 + C_{23} C_4 l_4 + C_{23} C_{45} l_5 - C_{23} S_{45} l_{5y} \\ S_4 l_4 + S_{45} l_5 + C_{45} l_{5y} \\ q_1 - S_2 l_2 - S_{23} C_4 l_4 - S_{23} C_{45} l_5 + S_{23} S_{45} l_{5y} \\ q_1 - S_2 l_2 \\ q_1 \end{bmatrix} \tag{28}$$

where $C_2 = \cos(q_2)$, $C_{23} = \cos(q_2 + q_3)$, $C_3 = \cos(q_3)$, $C_4 = \cos(\theta_4)$, $C_{45} = \cos(\theta_5)$, $C_5 = \cos(\theta_5 - \theta_4)$, $S_2 = \sin(q_2), \ldots, l_2$, l_4 and l_5 are the lengths of links 2, 4 and 5 and l_{5y} is the distance from joint 5 to the centre line of link 5 (Fig. 3).

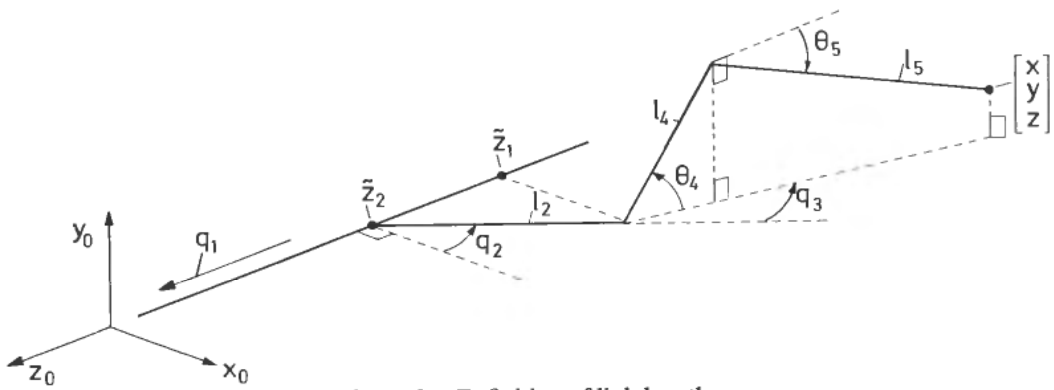

Figure 3. Definition of link lengths.

In this case, $J_A = \partial \boldsymbol{p}_A/\partial \boldsymbol{q}$ is singular for $q_2 = \pm\pi/2$, however, the manipulator was not expected to approach this position in our experiment.

The reference for z was used as reference for $\tilde{z}_1$ and $\tilde{z}_2$. In this way, the angles $\boldsymbol{q}_2$ and $\boldsymbol{q}_3$ were kept close to zero in the stationary case, which means that both joint 2 and 3 were moving around the middle of their working areas.

The augmented task space position reference was then

$$\boldsymbol{p}_{A,\,\mathrm{ref}} = [x_{\mathrm{ref}}, y_{\mathrm{ref}}, z_{\mathrm{ref}}, z_{\mathrm{ref}}, z_{\mathrm{ref}}]^T.$$

In § 4, the input generalized forces were

$$\boldsymbol{\tau} = M(\boldsymbol{q})J_A^{-1}(\boldsymbol{q})[\boldsymbol{u} - \dot{J}_A(\boldsymbol{q})\dot{\boldsymbol{q}}] - \boldsymbol{n}(\boldsymbol{q}, \dot{\boldsymbol{q}}) \tag{29}$$

The first term on the right side in (29) is the decoupling term, while the two last terms are the non-linearity compensation terms. In manipulator control, accuracy in the decoupling term $MJ_A^{-1}\boldsymbol{u}$ is important, while accuracy in the compensation of non-linear terms is not as critical to system stability (Egeland 1986, 1987; Spong and Vidyasagar 1985). We therefore omitted the two last terms in (29) in order to reduce the complexity of the controller algorithm. The input generalized force used was

$$\boldsymbol{\tau} = M(\boldsymbol{q})J_A^{-1}(\boldsymbol{q})\boldsymbol{u} \tag{30}$$

The servovalves had a bandwidth of about 600 rad/s, and this limited the bandwidth of the pressure feedback loop to 300 rad/s.

The sampling frequency was limited to 100 Hz by the A/D and D/A interface. This is sufficient for bandwidths up to 10 Hz. The load pressure feedback loops had analog controllers.

The algorithm was implemented on a Motorola 68020/68881 microprocessor, and the execution time of the total algorithm was 6 msec. This means that 60% of the capacity of the microprocessor was used. The algorithm was programmed in the high level language C, which made it easy to develop and modify the program.

The feedback gains were

$$g_i = \omega_i^2, \qquad i = 1, \ldots, 5 \tag{31}$$

and

$$g_{i+5} = 2\zeta\omega_i, \qquad i = 1, \ldots, 5 \tag{32}$$

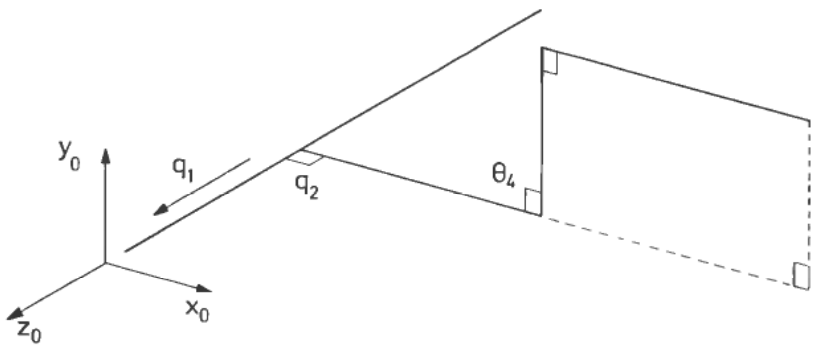

Figure 4. The zero position which was the starting position in the experiments.

where

$$\omega_i = \begin{cases} 20, & i = 1, 2, 3 \\ 7, & i = 4 \\ 5, & i = 5 \end{cases}$$

and $\zeta = 0{\cdot}25$. Friction and displacement flow gave a natural damping, and this is the reason for the low ζ.

The x and y references were selected as the x and y position of the hand when link 2, 3 and 5 are normal to the translational axis of joint 1, link 4 is vertical and link 5 is horizontal (Fig. 4). These references were kept constant.

First a step of 0·5 m in the z reference was given. As expected, the response of the end effector was fast with a time constant of about 0·25 s, while the positioning part had time constants of about 1 s (Fig. 5).

A saw-tooth reference is a more realistic reference for a spray-painting robot than a step change. The performance of the control system was considerably better for this type of reference than it was with a step change.

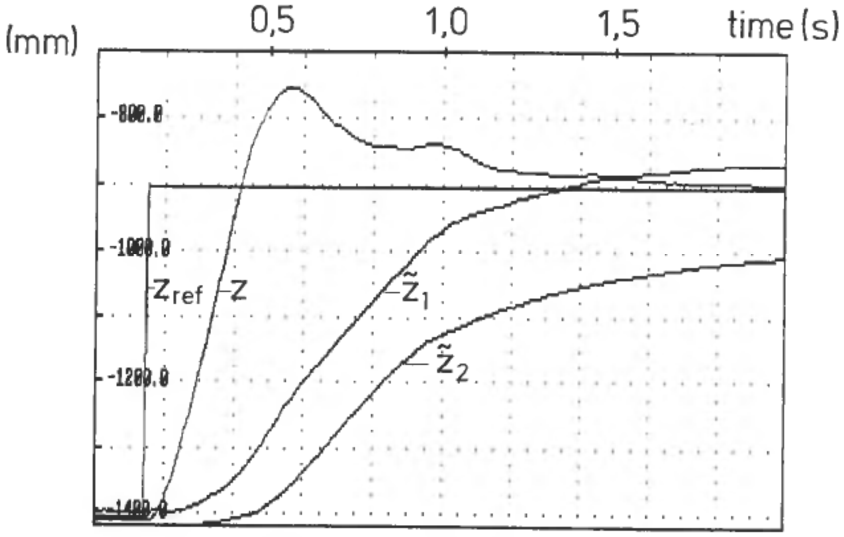

Figure 5. Experimental step response in the z_0 direction.

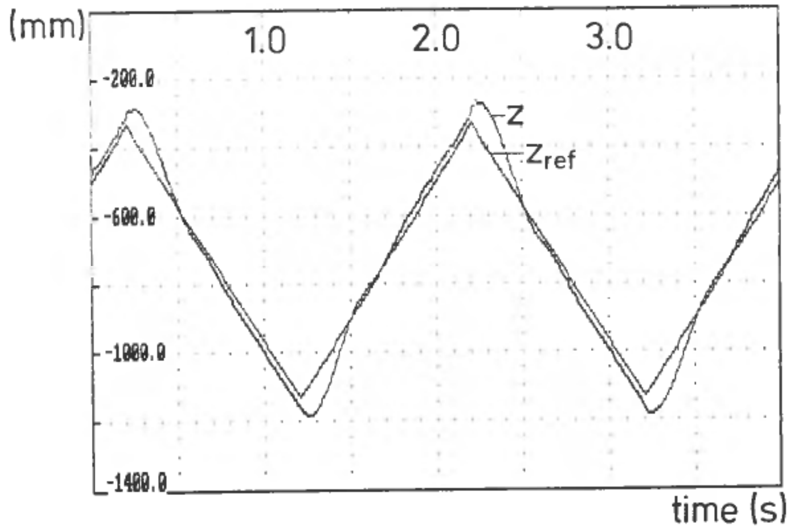

Figure 6. End effector tracking of saw-tooth reference with ± 80 cm/s.

The robot started from the zero position. The z_0 reference to z, $\tilde{z}_1$ and $\tilde{z}_2$ was a saw-tooth function where the velocity was ± 80 cm/s. The x_0 and y_0 reference to the end effector was constant. The period of the reference was 2 s. In this experiment, the closed loop undamped resonance frequency ω_i for the positioning of the end effector was increased from 20 rad/s to 30 rad/s. The end effector tracked the reference with a deviation of less than 3 cm in the z_0 direction except near the turning points (Fig. 6). The positioning part did not have high enough bandwidth to follow the reference (Fig. 7), but this was compensated by the outer manipulator.

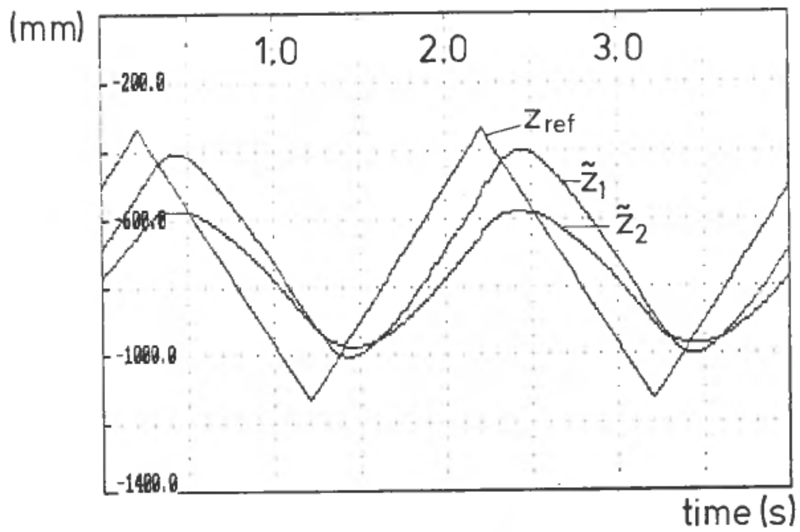

Figure 7. Position of positioning part.

6. Conclusion

A control system for the control of redundant manipulators consisting of a non-redundant manipulator mounted on a positioning part has been outlined. When this control system is used, low and high frequency motion is distributed to slow and fast parts of the manipulator. As long as there are no singularities close to the centre of the working range, singularities and the loss of degrees of freedom are avoided by selecting a suitable reference on-line, according to an *ad hoc* procedure.

With this control system, a robot can be given the same bandwidth as the outer manipulator, whereas with conventional control systems, the bandwidth is limited by the positioning part.

The control system was applied to an eight-link spray-painting robot where the five inner joints were controlled. The new control system gave the manipulator a bandwidth of about 20 rad/s, which compares favourably with conventional controllers where the bandwidth is limited to 5 rad/s. A further advantage with the new control system was that the inertia coupling was compensated for. Feedback linearization and decoupling were successfully implemented on this hydraulically driven robot using load pressure feedback to compensate for the actuator dynamics.

Future work on this control strategy for redundant manipulators should concentrate upon finding a more systematic means of selecting the reference to the positioning part. This could be achieved by optimizing a suitable performance index, which would enable collision avoidance to also be incorporated. Another interesting area is compliant motion for redundant manipulators, where the manipulator is in contact with the environment. Finally, manipulators with a very high degree of redundancy are worth investigating.

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