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A mathematical model for dynamic analysis of a flexible marine riser connected to a floating vessel

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A mathematical model for simulation of a flexible marine riser connected to a floating vessel is presented. The model is used for dynamic analysis of marine operations which include a riser and a floating vessel. The riser can be freely hanging from the vessel or fixed at the bottom. The vessel can be freely floating, moored by anchors or dynamically positioned. The performance of the riser model is evaluated by a comparison with results from a simulation study published by the American Petroleum Institute. Finally, a complete three dimensional, irregular sea-state analysis is presented to illustrate some of the capabilities of the simulation concept.

1. Introduction

A flexible marine riser is a long slender vertical pipe used in offshore drilling or production. The riser normally extends from a riser base to a floating vessel at the sea surface. It can contain a drill string and circulate drilling mud or return hydrocarbons from a production well. A typical situation is illustrated in Fig. 1.

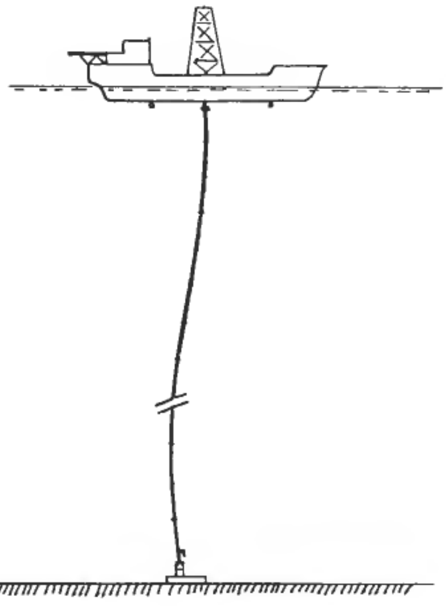

Figure 1. Vessel–riser operation.

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Flexible risers suspended on floating vessels are also used for other purposes such as gravel backfilling on pipelines and ocean mining.

In the North Sea today, flexible risers are operated in water depth greater than 300 m, and the depth still increases. New concepts for deepwater and early production are presented along with the exploration of new fields.

A vital component of all systems which are based on a floating production facility, is the long flexible riser. During operation, the riser will be subjected to both vessel motions and wave loads. Depending on the riser length, several modes of oscillation will be excited and for certain environmental conditions, significant bending stresses and fatigue can occur.

Therefore considerable effort has been put into the mathematical modeling and analysis of marine risers (Burke 1973, Morgan and Peret 1976, Larsen 1976, Gardener and Kotch 1976, Gone *et al.* 1975).

The effect of random waves and vessel motions (irregular sea state) has been analysed (Tucker and Murtha 1973, Sexton and Agbezuge 1976, Daring and Huang 1975, 1980) and procedures for field operations of marine risers are developed (Sheffield and Caldwell 1972, Childers and Martin 1980, Maison and Lea 1977). These studies have concentrated on the analysis of structural behaviour to predict stresses in the marine riser system.

As a consequence of the significant interest in the analysis of marine risers, the American Petroleum Institute has published a comparison of a number of riser simulation programs (API Bulletin 2J in January 1977). This publication presents the results from corresponding riser analysis produced by different computer programs.

In this paper, a mathematical model for the dynamic analysis of vessel–riser operations is described. By vessel–riser operations are meant all kinds of operations which include a floating vessel and a near vertical submerged pipe (riser). The riser can be fixed at the bottom or freely hanging from the vessel.

The complete simulation concept includes both a vessel and a riser model, however, discussing the dynamics of the riser primarily, only a brief review of the vessel model is included for the purpose of describing the most important excitation force of the riser system.

The riser model will be evaluated by a comparison with data published in the API Bulletin 2J, 1977. Finally, to illustrate some of the properties of the simulation concept, a complete three dimensional irregular sea-state analysis is performed for both a fixed and a freely hanging riser.

2. Mathematical models

2.1. *Marine riser*

The model of the marine riser is based on the general linear differential equation for a beam column with lateral loads in a vertical plane.

This analysis makes the basic assumption that transverse deformation is small in consideration of the total length of the riser. This small angle, large deflection theory is generally accepted valid within 10 degrees of deflection (Morgan and Peret 1976).

In addition it is convenient and reasonable to make the following assumptions:

Longitudinal motion of the riser is neglected.

The bending stiffness is constant along the riser pipe.

No coupling between the equations of motion in the two principal vertical planes (i.e. external pipes do not introduce structural or hydrodynamic coupling). If the pipe is axi-symmetric, the equations of motion in the two principal planes are identical.

$$\frac{\partial^2}{\partial z^2}\left(EI(z)\frac{\partial^2 x(z,t)}{\partial z^2}\right)-\frac{\partial}{\partial z}\left(T(z)\frac{\partial x(z,t)}{\partial z}\right)+M_m\frac{\partial^2 x(z,t)}{\partial t^2}=F(z,t) \tag{1}$$

$$\frac{\partial T(z)}{\partial z}=W_e \tag{2}$$

$M_m = M_{\text{riser}} + M_{\text{riser content}}$ (mass per unit length)

$W_e = W_{\text{riser}} + W_{\text{riser content}} - W_{\text{water}}$ (weight per unit length)

where

- $x(z, t)$ riser deflection
- z vertical coordinate
- t time
- $EI(z)$ bending stiffness
- $T(z)$ effective axial tension
- M_m total mass per unit length
- W_e weight per unit length of riser and content in water
- $F(z, t)$ total lateral force per unit length resulting from current and waves and other external forces
- M_{riser} Mass per unit length of riser pipe (steel pipe with additional lines, couplings, etc.)
- $M_{\text{riser content}}$ Mass per unit length of content in the pipe (drillpipe and hydrocarbons, etc.)
- W_{riser} air weight per unit length

The coordinate system is defined in Fig. 2. The models (1) and (2) describe the dynamics of a continuous near-vertical pipe (or stiff string) tensioned by the force T_e (effective tension) and loaded with the lateral force $F(z, t)$. T_e appears to be the tension of a riser buoyed up by the total amount of displaced water. According to this, tension of, for example, a drillstring will then add to the effective tension of the riser in the case of continuous contact between the drillstring and the riser wall. The true axial stress in the riser pipe is determined only by the pretension of the riser plus the air weight per unit length of the riser.

Boundary conditions. To solve the partial differential equation (1), a set of boundary conditions have to be defined. These conditions are determined by the specific riser operation to be analysed.

Actual operations are:

A free floating vessel with the riser suspended on the vessel and fixed at the bottom (i.e. drilling or production).

A free floating vessel with the riser suspended on the vessel and the lower end disconnected (i.e. gravel back-filling, re-entry to subsea installation, etc.).

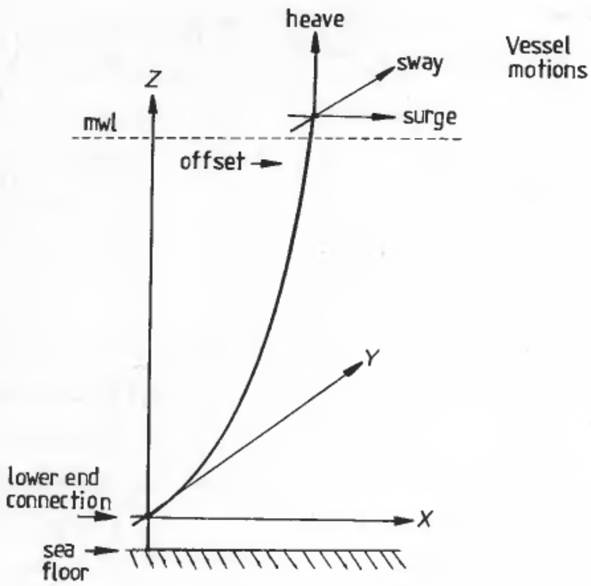

Figure 2. System coordinates.

The riser is fixed at the bottom with a free upper end (as in the case of a subsea loading terminal).

The boundary conditions are introduced by the derivatives

$$f^{(i)}(z)=\frac{\partial^{(i)}x}{\partial z^{i}} \tag{3}$$

where

$f^{(0)}(z)-x(z)$ displacement

$f^{(1)}(z)-\alpha(z)$ riser angle

$f^{(2)}(z)-M(z)$ moment

$f^{(3)}(z)-N(z)$ shear force

Normally, two conditions are known at each end of the riser. If the riser is freely hanging from the vessel, the boundary conditions are

$x(z=L)=$ vessel position

$M(z=L)=0$

$M(z=0)=0$

$N(z=0)=0$

$L=$ riser length

In this way the vessel motion will be introduced as a boundary condition and not as an external force. A necessary condition for this is that the vessel motions are not affected by the riser.

External forces. The lateral force $F(z, t)$ in eqn. (1) is due to hydrodynamic forces and concentrated horizontal forces (i.e. thrusters attached to the pipe).

The hydrodynamic force on the riser is determined from a modified form of Morrison's equation

$$F(z,t)=\frac{\rho_w C_D D}{2}\left|u(z,t)-\frac{\partial x(z,t)}{\partial t}\right|\left(u(z,t)-\frac{\partial x(z,t)}{\partial t}\right)$$
$$+\frac{\rho_w \pi D^2}{4}\left(C_M\frac{\partial u(z,t)}{\partial t}-(C_M-1)\frac{\partial^2 x(z,t)}{\partial t^2}\right) \quad (4)$$

where

- $u(z,t)$ water particle velocity due to current and waves as function of depth
- C_D drag coefficient
- ρ_w mass density of water
- D hydrodynamic riser diameter
- C_M inertia coefficient for a fixed cylinder in an accelerating flow

This equation is non-linear in the term $\left|u-\frac{\partial x}{\partial t}\right|\left(u-\frac{\partial x}{\partial t}\right)$.

The most difficult and least defined aspect of the analysis problem is the hydrodynamic flow around the riser. For a cylinder in a steady flow, the coefficients C_D and C_M are principally functions of Reynold's number with C_D approx. unity, and C_M approx. 1·5–2·0. For the wave case however (oscillatory flow), they appear to be functions of the Keulegan–Carpenter number N_{KC} which is defined

$$N_{KC}=u_{max}T/D$$

Here is u_{max} the maximum velocity and T the period of the oscillating mean flow. In this case C_D will be varying between 1 and 2 and C_M varying between 0·7 and 2·5 (Sarpkaya 1977). However due to the lack of theoretical description of the flow regime around a riser in an irregular sea state, a common approach to this problem is to use constant coefficients for calculating the hydrodynamic forces.

2.2. *Vessel motions*

The vessel motions are normally the major excitation of the marine riser. The vessel model is therefore a vital part of the total simulation concept. Depending on the operation (i.e. floating production platform, offshore loading, gravel backfilling) and the set of parameters to be studied (stress, deflection, position of riser, etc.), different levels of modeling are necessary for the simulation of vessel motions. Simulation of vessel motions for the type of vessel of interest (drillships, tankers, semisubmersibles, etc.) is usually based on the assumption that vessel motions can be separated into motions due to regular/irregular waves (i.e. first order vessel motions) and low frequency motions due to other external forces like current, wind, wave drift forces, etc. (i.e. second order vessel motions) (Langfeldt and Galtung 1976). For the simulation of vessel–riser operations, first order vessel motions are usually the most important with respect to the excitation of the riser dynamics. However, studying a gravel backfilling or a re-entry operation (dynamic positioning), second order vessel motions must be included.

The simulation of vessel motions are briefly described in the following sections.

Wave induced vessel motions. The wave induced vessel motions are calculated from a 'black box model', i.e. a set of transfer functions. These transfer functions can be calculated using strip theory (two-dimensional) or sink-source theory (three dimensional) or they can be measured in a model-tank test. The transfer functions describe the amplitude of the vessel motion response due to a regular wave component for different wave periods.

For a specific vessel heading, there is a transfer function for each degree of freedom (i.e. heave, pitch, roll, surge, sway and yaw).

To calculate the first order vessel motions due to a specific wave spectrum, the vessel motion response spectrum $R(\omega)$ is calculated from the relation

$$R(\omega)=|H(j\omega)|^2S(\omega) \tag{5}$$

$S(\omega)$ wave spectrum

$|H(j\omega)|$ magnitude of transfer function (vessel response amplitude relative to wave amplitude)

Hence the amplitude of a regular wave component of the vessel motion response is

$$a_r(\omega)=|H(j\omega)|a_s(\omega)$$

$a_s(\omega)$ real amplitude of regular wave

$a_r(\omega)$ real amplitude of vessel motion

The corresponding phase angle is

$$\psi(\omega)=\arg\,[H(j\omega)]$$

A synthetic time-history of the wave induced vessel motion (i.e. due to irregular waves) is now generated by summing regular wave components

$$y(t)=\sum_{j=1}^{J} a_r(\omega_j)\cos(-\omega_j t+\phi(\omega_j)+\epsilon_j) \tag{6}$$

$y(t)$ time history of vessel motion response

ω_j frequency of wave component

ϵ_j phase angle for each wave component determined from a uniform random number distribution between 0–2Π

j number of wave components

Low frequency vessel motions. Low frequency vessel motions are simulated by a special computer program developed for simulation of a general floating structure (Langfeldt and Galtung 1976). The structure can be freely floating, moored by anchors or dynamically positioned.

The equations of motion are in the matrix notation

$$(M+A)\ddot{x}+E(\dot{x})\dot{x}+B\dot{x}+DD_1(|\dot{x}|)\dot{x}+Kx=F_{ext}(t) \tag{7}$$

Here is:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} \begin{matrix} \text{surge} \\ \text{sway} \\ \text{heave} \\ \text{roll} \\ \text{pitch} \\ \text{yaw} \end{matrix}$$

The matrix $E(\dot{x})$ is derived by choosing the principle axis of the structure as the body fixed coordinate system.

In this case, the equations of motion of the structure (translation and rotation about the structure centre of gravity) can be written

$$M\ddot{x} + E(\dot{x})\dot{x} = F$$

Here is

M diagonal matrix containing the elements m, m, m, I_{11}, I_{22}, I_{33}

$$E(\dot{x}) = \begin{bmatrix} 0 & -mw_3 & mw_2 & 0 & 0 & 0 \\ mw_3 & 0 & -mw_1 & 0 & 0 & 0 \\ -mw_2 & mw_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & w_3(I_{33}-I_{22}) & 0 \\ 0 & 0 & 0 & 0 & 0 & w_1(I_{11}-I_{33}) \\ 0 & 0 & 0 & w_2(I_{22}-I_{11}) & 0 & 0 \end{bmatrix}$$

m total mass

I_{ii} component of the moments of inertia

w_i angular velocities

The damping force due to viscous drag is written

$$F_D = -DD(|\dot{x}|)\dot{x}$$

Here D represents the drag term (i.e. drag coefficients and projected area) and $D(|\dot{x}_2|)$ a diagonal matrix containing the absolute values of the instantaneous velocity vector $\dot{x}$ (i.e. $|\dot{x}_1|$, $|\dot{x}_2|$, $|\dot{x}_3|$, $|w_1|$, $|w_2|$, $|w_3|$). In addition are

A added mass force matrix

B linear potential damping force matrix

K restoring force matrix

$$F_{ext}(t)=\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{bmatrix} \begin{matrix} \text{surge} \\ \text{sway} \\ \text{heave} \\ \text{roll} \\ \text{pitch} \\ \text{yaw} \end{matrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \text{external forces} \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \text{external moments} \end{matrix}$$

The external forces and moments are due to wave drift forces, current, wind, mooring lines attached to the structure, hawser attached to the structure and thrusters.

The total vessel motions are calculated by adding the wave induced vessel motions (6) to the motions calculated by integration of (7).

2.3. *Environment*

The model of the environment includes wind forces on the floating structure, wave drift forces on the floating structure, wave elevation and water particle velocity due to wave motions and sea current as function of depth.

Wave model. Both wave elevation, wave induced vessel motions, wave drift forces and water particle velocity due to waves are derived from the wave elevation model.

Wave elevation records obtained from actual sea states are irregular and random in form. A wave record can be characterized by a wave energy spectrum calculated from the record.

The wave energy spectrum $S(\omega)$ is defined by

$$E=\rho g \int_0^{\infty} S(\omega)\, d\omega \tag{8}$$

Here is

- E total energy per unit area of the wave system
- ρ density of water
- g gravity constant

The simulation of waves is based on the linear superposition principle. Linear wave theory allows the summation of velocity potential, wave elevation, velocity and acceleration of j individual regular waves. The irregular wave can therefore be composed of a number of regular waves with random phase and frequency. Each regular wave component has an amplitude

$$\eta_{aj}=(2S(\omega_j)\Delta\omega)^{1/2} \tag{9}$$

- ω_j frequency of wave component
- $\Delta\omega$ frequency width of wave component

The wave elevation of a synthetic irregular wave is then

$$\eta(x, t) = \sum_{j=1}^{J} \eta_{aj} \cos(k_j x - \omega_j t + \epsilon_j) \tag{10}$$

x horizontal coordinate

k_j wave number

ϵ_j phase angle for each component determined from a uniform random number distribution between 0° and 360°

j number of wave components

Using linear wave theory and the deep water assumption (i.e. circular water particle motion), the horizontal component of the water particle velocity and acceleration along the riser profile (to be used in eqn. (4)), are calculated from the derivatives of eqn. (10).

3. Numerical methods

Method of finite differences

The model established in eqn. (1) is a non-linear fourth order partial differential equation. To solve this numerically, the model is transformed to a set of first order differential equations by means of an explicit difference method.

The following finite difference scheme is used:

$$\frac{\partial^2}{\partial z^2}\left(EI\frac{\partial^2 x(z,t)}{\partial z^2}\right)\bigg|_{z=z_n} \approx \frac{EI}{\Delta z^4}(x_{n+2} - 4x_{n+1} + 6x_n - 4x_{n-1} + x_{n-2}) \tag{11}$$

$$\frac{\partial}{\partial z}\left(T(z)\frac{\partial x(z,t)}{\partial z}\right)\bigg|_{z=z_n} \approx \frac{1}{4\Delta z^2}(T_{n+1}x_{n+2} - (T_{n+1} + T_{n-1})x_n + T_{n-1}x_{n-2}) \tag{12}$$

The approximation of the derivatives by finite differences, means that the riser pipe is divided into N equally spaced elements. The element length is Δz.

Introducing the external force (4), the equations of motion at node n can be written (x-deflection)

$$\left.\begin{aligned} \dot{x}_n &= p_n \\ (M_m + (C_M - 1)\rho_w \pi D/4)\dot{p}_n &= a_n x_{n-2} + b_n x_{n-1} + c_n x_n + d_n x_{n+1} + e_n x_{n+2} \\ &\quad + \tfrac{1}{2}\rho_w C_D D((u_{xn} - p_n)^2 + (u_{yn} - q_n)^2)^{1/2}(u_{xn} - p_n) + \tfrac{1}{4}\rho_w C_M a_{xn} \end{aligned}\right\} \tag{13}$$

The equations of motion for the y-deflection will be analogous. Here is:

$$a_n = T_{en-1}/4\Delta z^2 - EI/\Delta z^4$$
$$b_n = d_n = 4EI/\Delta z^4$$
$$c_n = -(T_{en+1} + T_{en-1})/4\Delta z^2 - 6EI/\Delta z^4$$
$$e_n = T_{en+1}/4\Delta z^2 - EI/\Delta z^4$$

The effective tension at node n, T_{en}, is calculated directly from eqn. (2).

The elements a, b, c, d, e, at node 1, 2 (lower end) and N, $N+1$ (upper end) will be modified according to the boundary conditions introduced as described in § 2.1.

Now the final model can be established on the basis of eqn. (13) using matrix notation

$$\left.\begin{aligned} \dot{x} &= p \\ M\dot{p} &= Kx + HU(v_x - p) + Aa_x \\ \dot{y} &= q \\ M\dot{q} &= Ky + HU(v_y - q) + Aa_y \end{aligned}\right\} \quad (14)$$

Here is

- x, y riser coordinates (profile)
- p, q riser velocity
- v_x, v_y water particle velocity (horizontal component)
- a_x, a_y water particle acceleration (horizontal component)
- M mass coefficients (diagonal matrix) (includes added mass)
- K stiffness coefficients (band matrix)
- H drag coefficients (diagonal matrix)
- U water particle velocity relative to riser (diagonal matrix)

The elements of U are calculated from

$$|U_{rel}|_n = ((u_{xn} - p_n)^2 + (u_{yn} - q_n)^2)^{1/2}$$

A acceleration force coefficient (diagonal matrix)

Though the equations of motion for the two principle vertical planes are identical with respect to the mass and structural terms (stiffness matrix), a three dimensional model is necessary to include the effect of different angles of attack of current, waves and vessel motions. This effect is taken into account through the modeling of water particle velocity and acceleration due to waves and current and finally through the calculation of the hydrodynamic force using relative velocities.

Method of integration. The model established in eqn. (14), includes $N-1$ independent complex eigenvalues. The corresponding eigenvectors are recognized as the modes of riser oscillation. Hence the number of modes of oscillation which are modeled, are one less than the number of elements. This means of course that two elements are necessary to model the first mode of oscillation, and so forth.

The method of integration of eqn. (14) is important to the speed and stability of the simulation. Due to the complex eigenvalues of the model and the small damping of the highest modes of oscillation, an implicit or a higher order explicit method will be necessary. In the following examples a fourth order Runge–Kutta method is used. This method will be conditionally stable and the step length will be related to $(\Delta z)^n$. The power n depends on whether the riser stiffness matrix K is dominated by bending stiffness (EI) or tension (T). Bending stiffness leads to the strongest condition.

4. Simulations

4.1. *Evaluation of the riser model*

The riser simulation model is evaluated using data published in the API Bulletin: 'Comparison of Marine Drilling Riser Analysis', 1977. This publication presents results from the simulation of some standard cases performed by 9 different riser programs. The result from each program is presented, and no attempt is made to define the 'best' result. However, a comparison with these data gives at least a general impression of the validity of the riser model described in the paper.

In the following are presented some results from the simulation of these standard cases using the mathematical model described in the preceding sections. In the same figures are indicated an approximate 'range' of values at different depths as read from the corresponding API curves. This range (the diameter of the circles) is derived by neglecting the most extreme values and approximately measuring the spread of the remaining values.

Description of cases. The riser analysed is assumed to be two dimensional, with all forces and elements lying within a vertical plane. The riser contains mud only and is connected to a frictionless ball-joint at the upper and lower end. Waves are assumed to act in the direction of the positive offset, and the upper end is exposed to horizontal vessel motions. Only regular wave analysis is performed, and the horizontal motion of the vessel (surge) is sinusoidal with the same period as the wave. Top tension is constant. Figure 2 shows the typical riser configuration.

Specification of main data

Vertical distances	
Mean water level to riser support ring	15·24 m
Seafloor to lower ball-joint	9·14 m
Riser data	
Riser pipe, outside diameter	0·4064 m
Riser pipe, inside diameter	0·3747 m
Modulus of elasticity E	207×10^9 N/m^2
Weight, 50 ft riser joint, complete in air	$38{\cdot}36 \times 10^3$ N
In seawater	$32{\cdot}426 \times 10^3$ N
Densities	
Drilling mud	$1{\cdot}44 \times 10^3$ kg/m^3
Hydrodynamic force constants	
Drag coefficient C_D	0·7
Mass coefficient C_M	1·5
Effective diameter for wave forces	0·6604 m
Environment	
Wave height (peak to peak)	6·1 m
Wave period	9 s
Excitation	
Vessel surge amplitude (peak to peak)	1·22 m
Vessel surge phase angle	15°
(Phase angle of peak vessel surge after wave crest)	

A dynamic analysis is performed for the following cases

Case	500-20-1-D	500-20-2-D	1500-20-1-D	1500-20-2-D
Water depth (m)	152·4	152·4	457·2	457·2
Riser length (m)	158·5	158·5	463·3	463·3
Top tension (10^3 N)	534·0	890·0	1290·5	2225·0
Static offset (m)	4·57	4·57	13·7	13·7

The following parameters are used for the difference approximation (i.e. space/time differences)

Case	500-20-()-D	1500-20-()-D
No. of riser elements	25	30
Length of riser element (m)	6·33	15·44
Time increment (s)	0·05	0·05

The use of an explicit method of integration leads to a relatively short time increment due to the large number of eigenvalues modeled.

Results of the dynamic analysis. Performing a regular wave analysis (harmonic excitation), time integration will produce a steady state solution after a short transient period.

The results from this steady state motion of the riser are described by the envelopes (maximum and minimum values) of the riser deflection and bending stress. The results from the simulations are shown in Figs. 3–6. As expected, the deflection profile, converges to the amplitude of oscillation of the vessel motion. Bending stress is computed on the convex side of the riser circumference. As previously mentioned

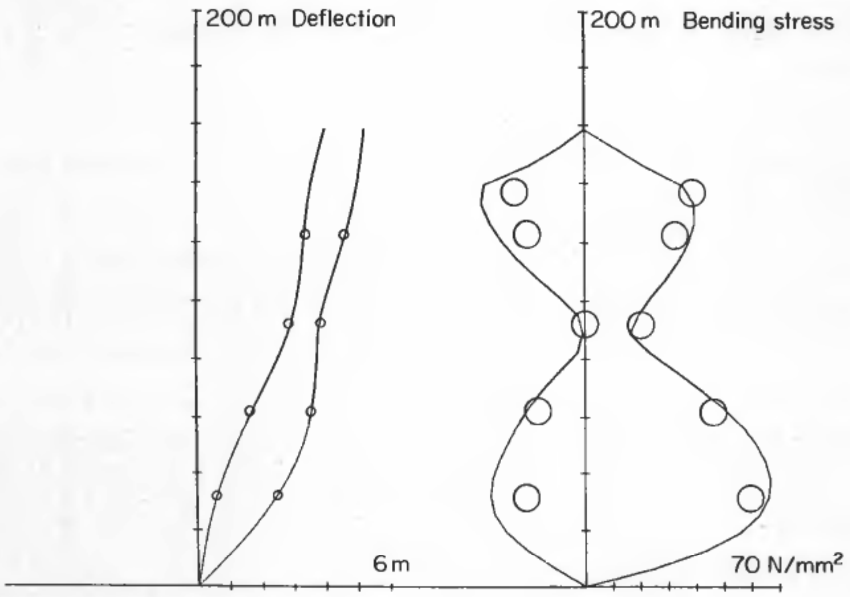

Figure 3. API simulations, Case 500-20-1-D.

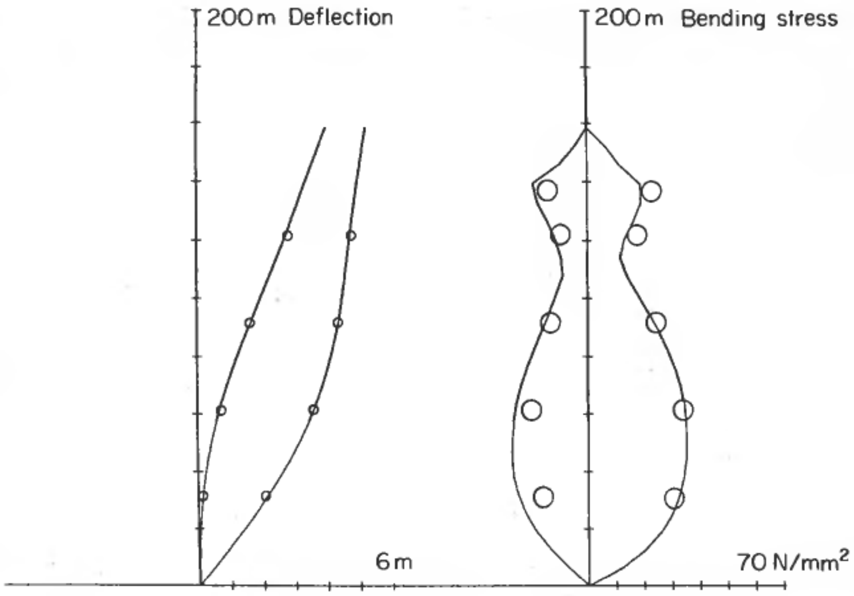

Figure 4. API simulations, Case 500-20-2-D.

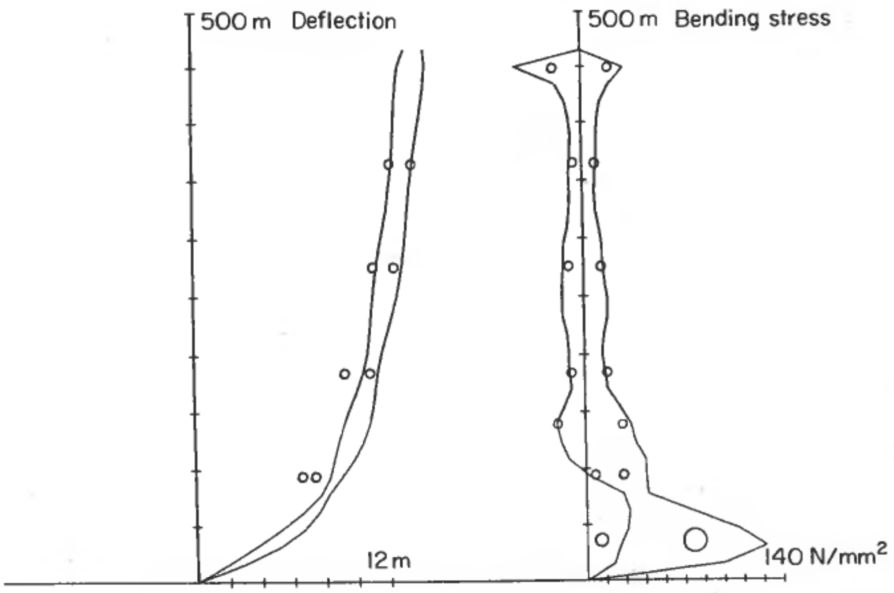

Figure 5. API simulations, Case 1500-20-1-D.

results as read in the corresponding API curves are indicated as circles in the figures. The results show a good agreement with these data.

4.2. *The freely hanging riser*

To illustrate some properties of the riser model described, a simulation of the specific riser in the preceding section is performed with a free lower end (i.e. the riser

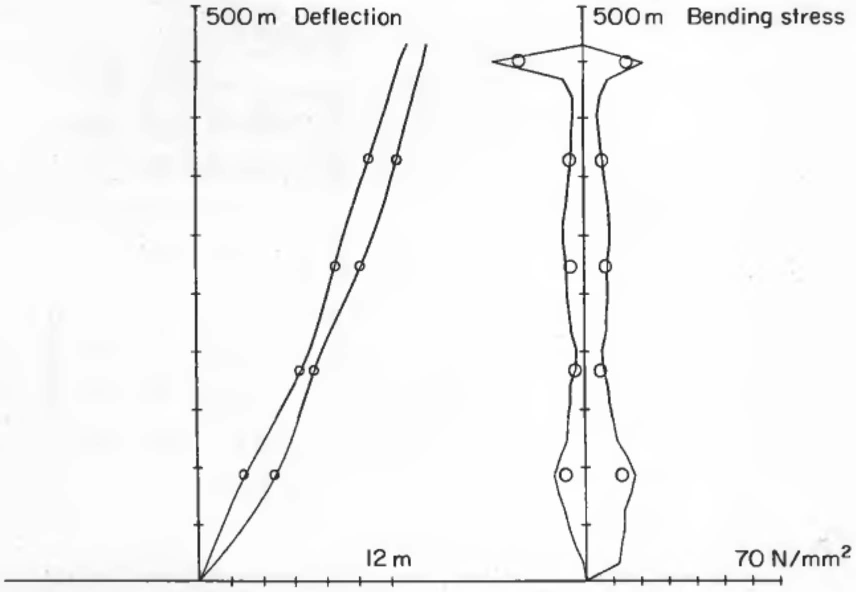

Figure 6. API simulations, Case 1500-20-2-D.

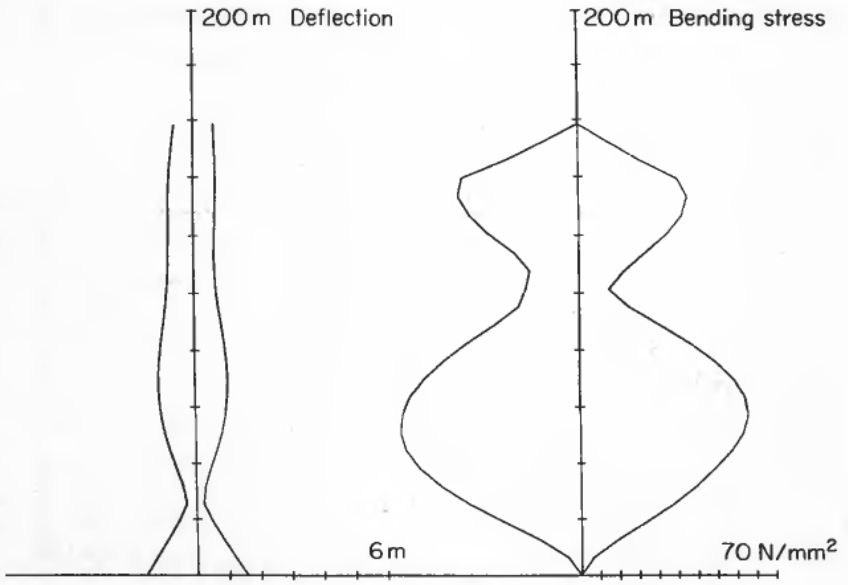

Figure 7. Freely hanging riser 500 ft.

is disconnected from the riser base at the seabed). In addition to the free lower end, the riser is now assumed to be filled with water instead of drilling mud. The top tension will in this case be equal to the weight of the submerged riser.

The results of the regular wave analysis described in the preceding section (the same excitation as before), are shown in Fig. 7 (500 ft riser) and Fig. 8 (1500 ft riser). As indicated, the modes of excitation are much the same as the modes of the low tensioned fixed riser. However, the amplitude of the bending stress has increased.

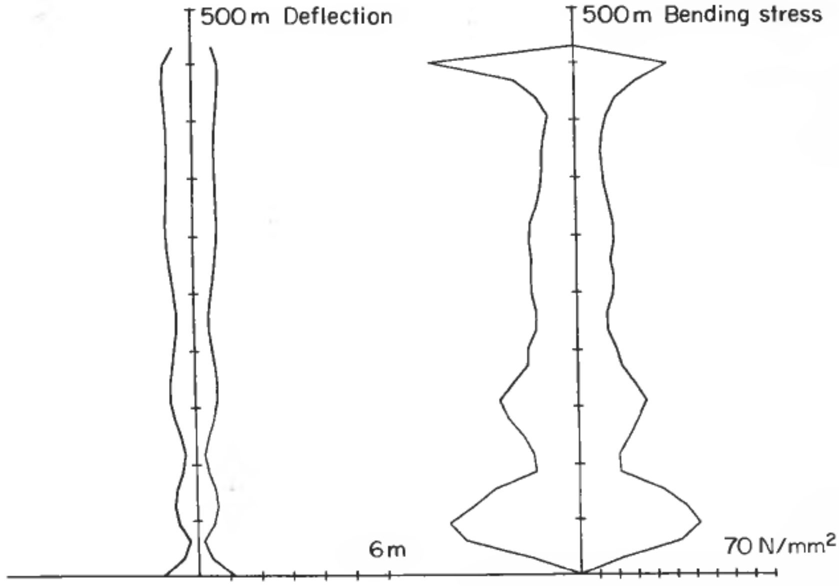

Figure 8. Freely hanging riser 1500 ft.

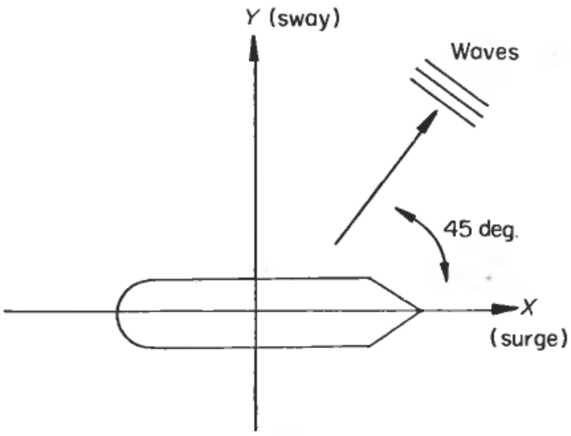

Figure 9. Vessel coordinates.

4.3. *Irregular wave analysis*

Finally, a complete three dimensional, irregular sea-state analysis is performed for the 500 ft API riser. In this case the riser is connected to a conventional drillship (length: 135 m, deplacement: approx. 20 000 tons). The wave direction is 45° on stern as indicated in Fig. 9, and the irregular sea state is defined by the parameters

Significant wave height: 4 m

Mean wave periods: 8 s

As described in § 2.3, the time history of the wave elevation is generated on the basis of a Pierson–Moskowitz wave spectrum shown in Fig. 10. The number of wave

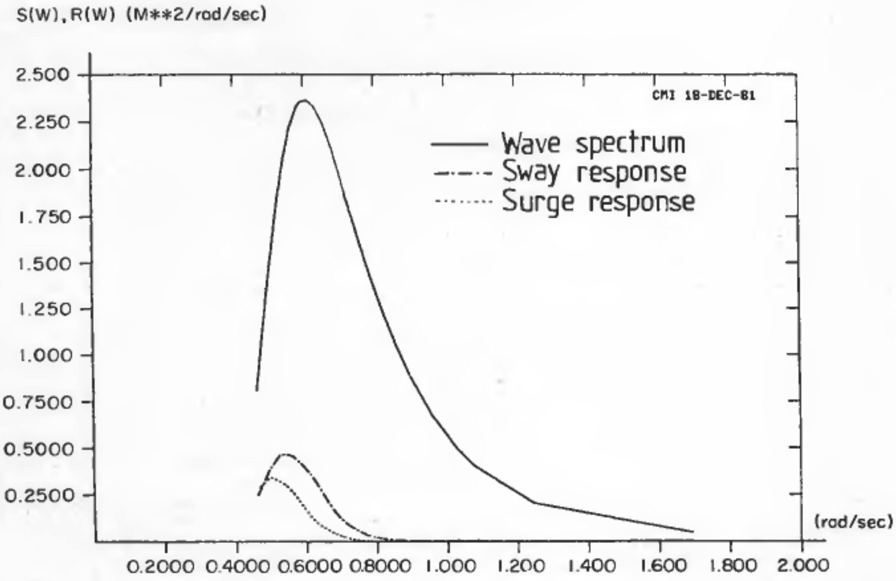

Figure 10. Wave and vessel motion response spectrum.

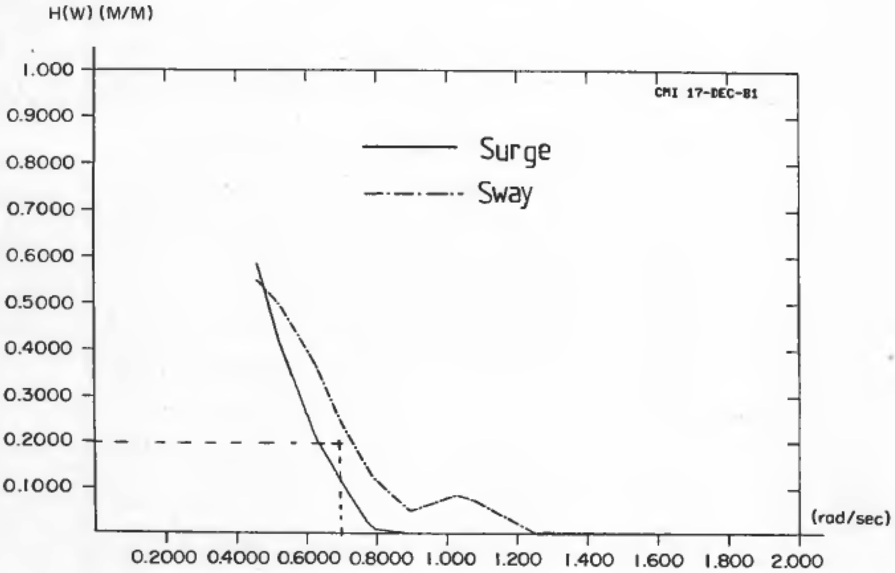

Figure 11. Surge and sway operators (amplitude).

components used in this example is 25. The horizontal water particle velocity and acceleration along the depth are calculated on the basis of linear wave theory.

The wave induced vessel motions (surge and sway motions) are calculated from the transfer functions for the vessel as described in § 2.2. These transfer functions (amplitude and phase angle) are shown in Figs. 11 and 12, and the surge and sway response spectra corresponding to the given sea state are indicated in Fig. 10. The resulting time history of the surge and sway motion is shown in Fig. 13.

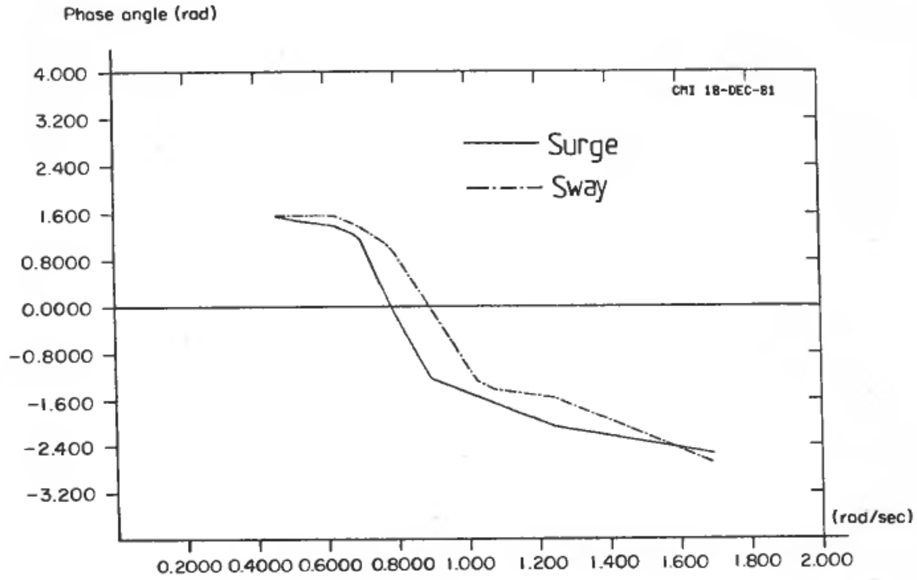

Figure 12. Surge and sway operators (phase angle).

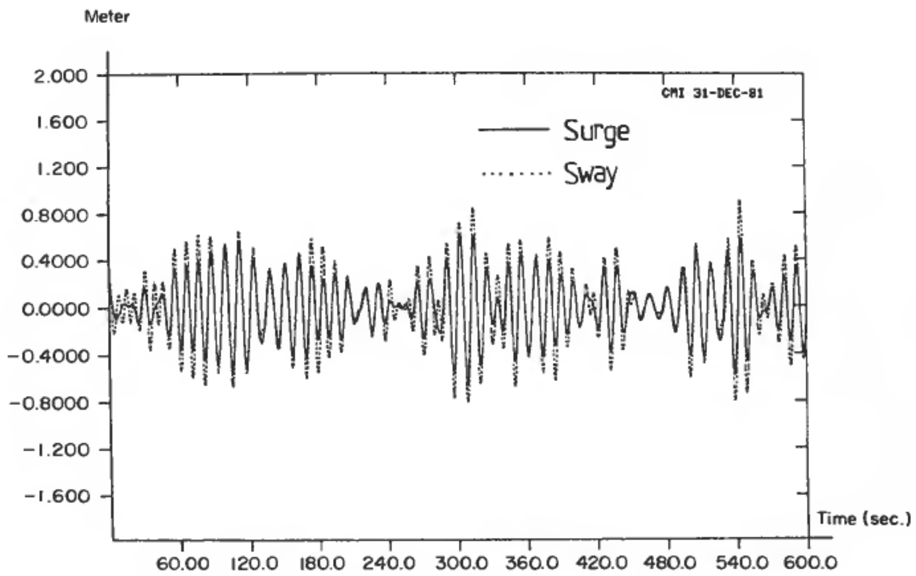

Figure 13. Time history of surge and sway.

The 500 ft API riser is now excited by the irregular waves and vessel motions for the following cases:

Fixed riser, top tension = 534×10^3 N (low)

Fixed riser, top tension = 890×10^3 N (high)

Free lower end

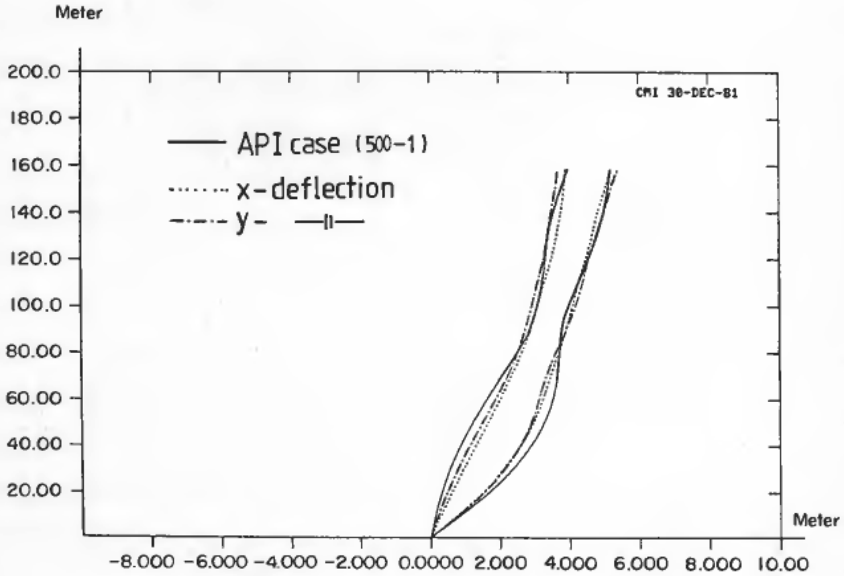

Figure 14. Riser deflection, fixed riser, 500 ft, low tension.

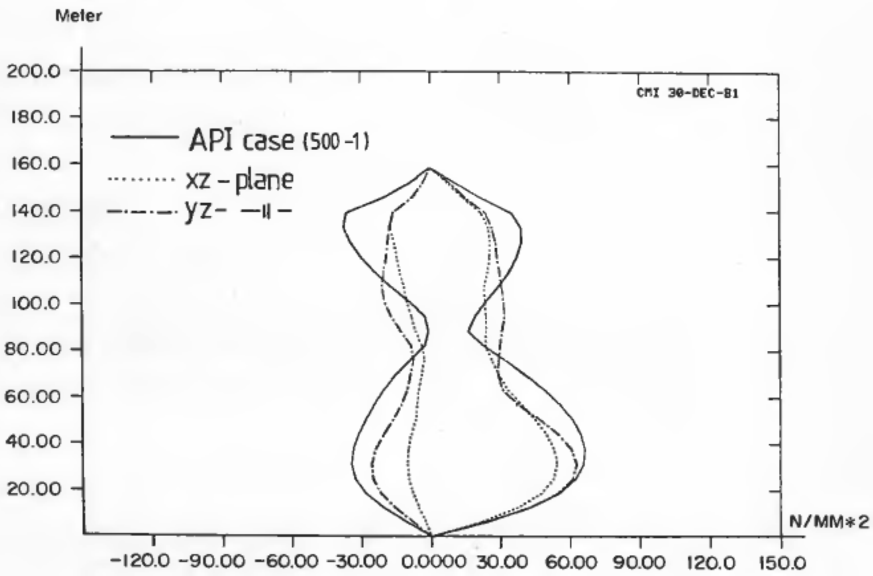

Figure 15. Bending stress, fixed riser, 500 ft, low tension.

The riser response (deflection and bending stress) is calculated for the *x*- and *y*-deflection simultaneously. Note that, though the models of the riser structure in the *xz*- and *yz*-plane are identical, they are coupled through the hydrodynamic force term (Morisons' equation).

Fixed riser, irregular sea-state. The static offset of the riser is the same as for the previous 500 ft riser cases. Time histories of riser deflection and bending stress are obtained by integration of the numerical riser model.

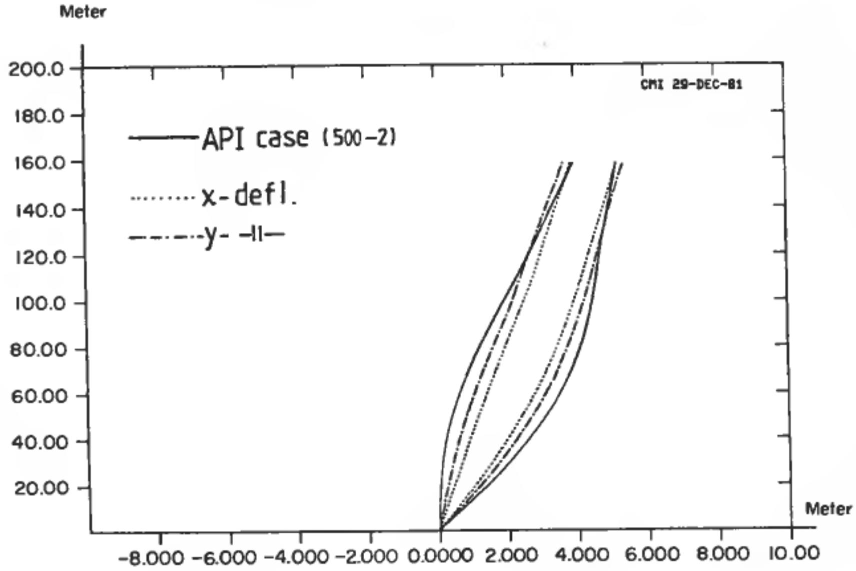

Figure 16. Riser deflection, fixed riser, 500 ft, high tension.

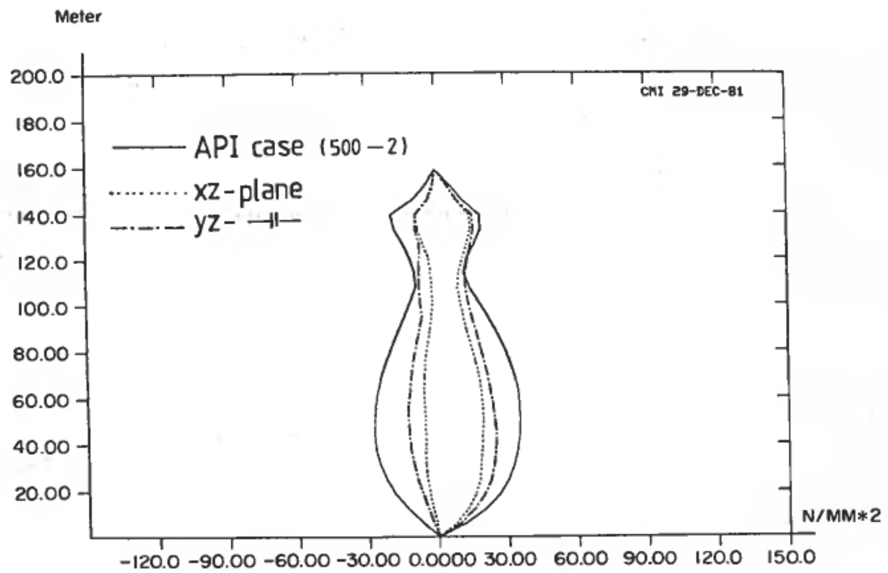

Figure 17. Bending stress, fixed riser, 500 ft, high tension.

The envelopes of riser deflection and bending stress are calculated on the basis of these time histories as shown in Figs. 14–19. In these figures are also indicated the corresponding envelopes from the *regular* wave analysis in § 4.1. The ratio between the wave amplitude and the vessel motion amplitude for the *regular* wave case (API-simulations), is indicated in Fig. 11. Though this comparison is somewhat artificial, it indicates the difference between a regular and an irregular wave analysis of a marine riser. In the irregular sea state situation, the riser will never reach a steady state

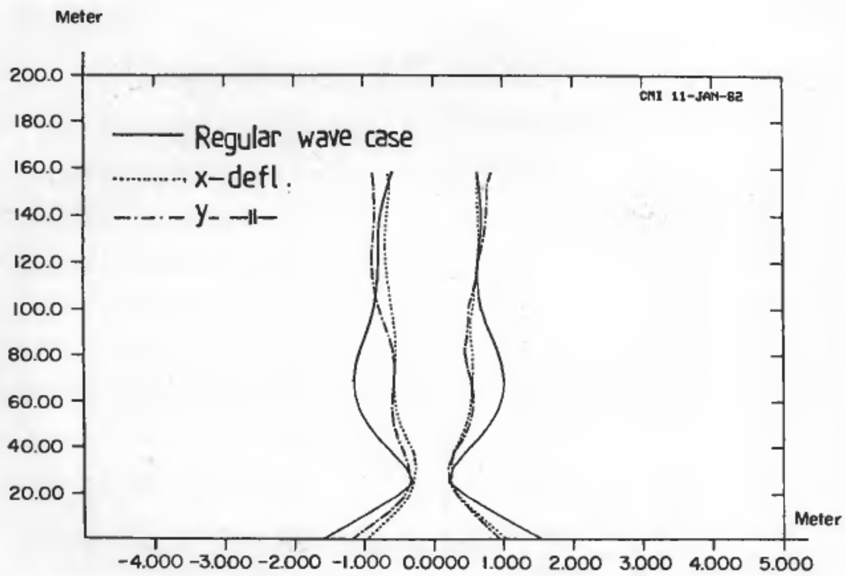

Figure 18. Riser deflection, freely hanging riser, 500 ft.

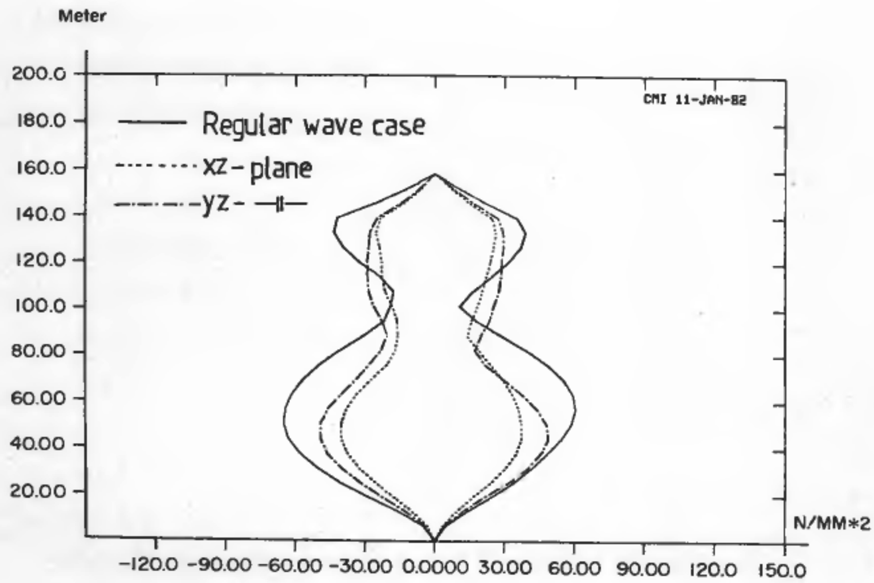

Figure 19. Bending stress, freely hanging riser, 500 ft.

oscillation. Due to the non-linear term in the riser model, several modes of oscillation will be excited simultaneously in the transient region. Depending on the energy of the excitation, a *regular* wave analysis might lead to conservative results with respect to bending stress compared to an *irregular* wave analysis.

Freely hanging riser. Irregular sea-state. The irregular sea state analysis is also performed for the freely hanging riser. The results are shown in Figs. 18 and 19. As for

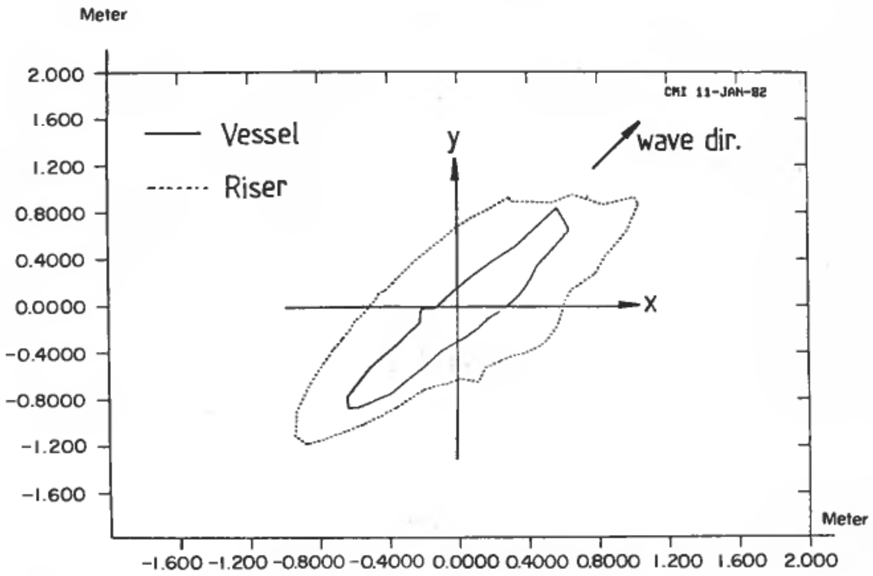

Figure 20. Envelopes of $X7$ motion (vessel and lower riser end).

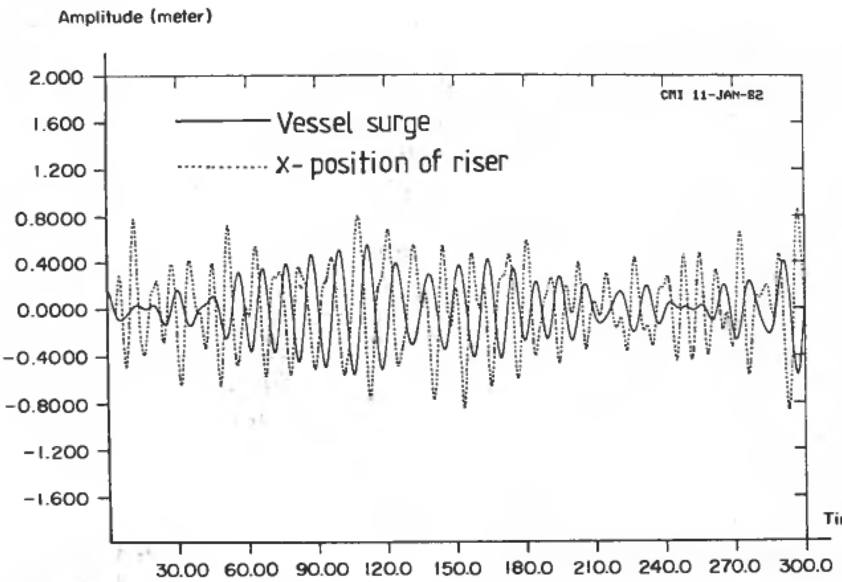

Figure 21. Time history of vessel surge and lower riser end.

the preceding cases the results from the *regular* wave analysis are also indicated in the figures.

The most interesting observation is the amplification of the vessel motions at the lower end of the riser. This is clearly illustrated in Fig. 20 showing the envelopes of the horizontal position of the vessel and the corresponding position of the lower riser end.

Figure 21 shows the time history of the surge motion of the vessel compared to the corresponding motion of the lower riser end. Though there is a correlation at a certain time lag, the motion of the riser will mainly be determined by the modes of oscillations excited by both the waves and the vessel motions.

The motion of the lower end of a freely hanging riser will be a limiting factor for a re-entry operation of a drilling or production riser to a subsea wellhead or manifold centre. If such re-entries have to be performed frequently, for example during production from a floating vessel in hostile waters or offshore loading directly from the seabed, the motion of the lower end could be controlled using a set of horizontal thrusters mounted on the riser. However, in the case of a steel riser, significant power would be required to compensate for the bending and inertial forces at the lower end of the riser.

5. Conclusion

A mathematical model describing the dynamics of a flexible marine riser connected to a floating vessel has been presented. The riser can be freely hanging or fixed at the bottom, and the vessel can be freely floating, moored by anchors or dynamically positioned.

The model is suitable for dynamic analysis of operations, such as deep water drilling using a DP-vessel, production from a floating unit, re-entry to a subsea installation with a freely hanging riser, continuous gravel back-filling on pipelines through a fall-pipe, etc.

The comparison with the results from the API study, indicates reasonable predictions by the riser model, though with some greater mean deflection for the low tensioned 1500 ft riser. The three dimensional, irregular sea state analysis shows some of the capabilities of the simulation program. The results are in this case presented as envelopes of computed profiles within a specified time. This is usually the most interesting for a design analysis. However, an interactive simulation can easily be arranged to study the dynamics of a complete operation (for example a re-entry of a riser to a subsea installation). The simulation concept is basically developed for the simulation of global dynamics of complex operations. There is practical limitation with respect to the resolution of forces and computed stresses (i.e. limited number of elements). However, this limitation is usually not serious.

Acknowledgment

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