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A small econometric model of Norway

ROLF HENRIKSEN† and AAGE THUNEM‡

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This paper presents a small model of the Norwegian economy which has been developed by the Division of Engineering Cybernetics at the Norwegian Institute of Technology. It was primarily designed to perform ex-post simulations of the overall behaviour of the economy during the period 1968–77. The experiments show that a cut in central government income taxes might have strengthened the economy significantly without increasing inflation. They thus indicate that a high level of personal income taxation, as in Norway today, may be highly inflationary, as opposed to what is asserted in many textbooks.

1. Introduction

The use of econometric models for analysing the behaviour of the overall economy has become widespread in the last two decades, especially in the U.S. (see Burmiester and Klein 1974). Econometric models—of which there are now a considerable number all over the world—have been used for forecasting, estimating the quantitative impact of government policies, and testing various hypotheses about the structure of national and sectoral economies.

The model presented in this paper consists of approximately 70 equations and was primarily designed as a simulation instrument to perform quantitative analyses of the behaviour of the Norwegian economy, during the period 1968 through 1977 for which the equations were fitted. It should be stressed that the model below is experimental and represents a highly preliminary stage of modelling the Norwegian economy.

Government planning, forecasting, and budgeting are, in Norway, primarily carried out by extensive use of two large-scale models, the MODIS and MSG models. The MODIS model (Sevaldson 1971, Bjerkholt and Longva 1974, 1977) is a short- and medium-term model used for, among others, preparation of the annual national budgets, and short- and medium-term planning. The MSG model (Johansen 1974, Lorentsen and Skoglund 1976) is a multisectoral growth model used for long-term planning and forecasting. The so-called Aukrust or PRIM model (Aukrust 1970) is a model for price and income determination of an open economy. This model is today integrated as part of the MODIS model.

Both of the above-mentioned models are equilibrium models which express equilibrium relations between the variables, e.g. between growth rates as in the MSG model. They contain, it may be claimed, no true dynamics which have to be incorporated in order to explain and account for relevant inertia in the working of the economy. Although static models may serve as a useful first approximation to explain

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† The University of Trondheim, The Norwegian Institute of Technology, Division of Engineering Cybernetics, Norway.

‡ SINTEF, The Foundation of Scientific and Industrial Research at the Norwegian Institute of Technology, Trondheim, Norway.

and predict the effect of certain exogenous variables on the endogenous variables, it is a well-known fact, which has been stressed by many economists, that the neglect of relevant inertia in static models may often lead to wrong conclusions and predictions. For example, exports of goods and services are taken to be exogenous in the MODIS model, whereas the MSG model contains a variable called net exogenous demand (government demand+exports−imports of competitive goods). This may be justifiable in a short-term model, but will be highly questionable in a long-term model since exports certainly must depend upon wages and productivity in the long run. Similar comments can be made about investments which are treated exogenously in the MSG model.

When we started to build the model, there were two features which we felt had to be retained whatever the changes that were decided upon later on. First, the model would have to be dynamic in order to reflect important inertia. Second, an important aim was that no variables which meaningfully could be treated as endogenous should be taken to be exogenous. This does not mean that all of the variables which actually are treated as exogenous are completely independent of the endogenous variables. However, the structure of the model, e.g. the aggregative form, may not allow any meaningful models of these variables, or they can be treated as exogenous as long as certain constraints are not exceeded. For example, government consumption expenditures are usually taken to be exogenous although the government certainly has some budget constraint. However, the government does not have to run a balanced budget; it can always allow a certain deficit simply by issuing bonds and printing money. As long as government expenditures are within realistic limits we can therefore treat them as being exogenous.

2. Structure of the model

The model may conveniently be divided into five groups of equations which explain (1) components of GNP; (2) prices, wages, and employment; (3) capital, assets, and liabilities; (4) distributive shares; and (5) miscellaneous variables. A brief discussion of these groups will be given.

Components of GNP

The gross national product (GNP) consists in this model of six components: personal consumption expenditures, gross fixed investments, inventory changes, exports, imports, and government consumption expenditures. Since the latter are taken to be exogenous, there are actually only five equations in this group.

Personal consumption expenditures are assumed to be a function of the previous year's expenditures, the current year's private disposable income, and the current year's price deflator. A similar type of consumption function is used in the MODIS model (see Biörn 1974).

Gross fixed investments have been split into four parts: depreciation, net non-residential investments except investments in the crude oil production sector, net residential investments, and net investments in the crude oil production sector which are treated exogenously. Depreciation is taken to be a function of the fixed capital stock. Non-residential and residential investments are modelled using some of the income components as explanatory variables. In more sophisticated econometric models, investment behaviour is explained using, among others, prices, interest rates, wage rates, and user costs of capital as explanatory variables. It is, however, a well-known fact that business investments are among the hardest things to model in economics. Several of the presumably relevant explanatory variables turn out being of little or no importance when investment data are being analysed. Although our investment model is rather crude, it performs quite well in the long run. Total simulated investments turned out to be very close to real investments in the period 1968–77.

Inventory changes have been modelled using total private sales, export changes, import changes and the inventory stock as explanatory variables.

The export model is based upon the previous year's exports, world production, and relative prices as explanatory variables. The model contains both demand and supply terms. The demand terms consist of a world production index, and the ratio between export prices (multiplied by the foreign exchange rate) and foreign prices (using import prices as a proxy for this). The supply term consists of the ratio between export and domestic prices.

The import equation is a pure demand model using previous year's imports, domestic demand, and the ratio between import and domestic prices as explanatory variables.

Prices, wages and employment

Domestic prices are in the model primarily represented by the GNP implicit price deflator. This deflator turns out to be quite a good approximation of the personal consumption implicit price deflator, and a fair approximation of the fixed investments implicit price deflator, although it is somewhat too low towards the end of the period. The deviations from the government consumption implicit price deflator, however, turn out to be significant, so this price deflator has been modelled separately. The implicit price deflators for inventory changes, exports, and imports are all exogenous in the model.

Inflation is assumed to arise from four sources: increased indirect taxes, wage increases, increased import prices, and the price-taking behaviour of producers. The model for the price-taking behaviour of producers has been inspired by the following. Assume a technology where output Q is determined by a two-factor Cobb–Douglas production function with constant returns to scale, viz.

$$Q = aK^{\alpha}L^{1-\alpha} \tag{1}$$

where K is capital, L is labour and a and α are constants ($0<\alpha<1$). Given a constellation between the wage rate w and profit rate, or return to capital, ρ, and assuming that production is based on profit maximization on a competitive economy, it can be shown that the Cobb–Douglas production function will generate a linear expansion path, i.e. the ratio between capital and labour remains constant. Furthermore, the share of labour (and hence of capital) in total output remains constant (see Allen 1968, or Branson 1972, pp. 380–391). Now, we have in our model used a production function of the form

$$Q(k) = aK^{\alpha}L^{1-\alpha}\gamma^{k} \tag{2}$$

where the last term is used to account for technical progress. Subject to the same assumptions as previously, it can be shown that the shares of labour and capital in total output still remain constant, but the ratio between w and ρ grows at rate γ

(see Branson 1972, pp. 391–395). In our price model we assume producers to know the fact that the share of labour S_L is constant. This share is computed to be

$$S_L = \frac{WL}{L^w Q} \tag{3}$$

where W is aggregate wages while L^w is wage and salary employment.

Government consumption expenditures consist mainly of services. The implicit price deflator for this GNP component is therefore strongly related to the wage rate, and it has been modelled using the GNP price deflator and the wage rate as explanatory variables.

Aggregate wages, i.e. total wage and salary disbursements (in billions of Nkr), have been modelled using a net, i.e. after tax, real wage model where price anticipations (or expectations) are allowed to play a part. Price expectations, α_6, are taken to be exogenous although they generally must be a function of the state of the economy. We believe, however, that what people anticipate about future prices is for a great deal based on what forecasters, e.g. the Government, predict. If the forecasters agree to lie deliberately about what they believe future prices to be, it may affect people's anticipations because very few have an econometric model with which the forecasts can be checked. As long as the expectations are within reasonable limits we may therefore treat them as being exogenous. Total compensation of employees is computed from aggregate net wages by adding central government taxes, municipal taxes, social security premiums, and employers' contributions to social security schemes.

Total employment (man–years) is computed by linearizing the estimated Cobb–Douglas production function (eqn. (2)) about the simulated historical paths for real GNP and employment. Doing this, we find

$$\Delta L(k) = \frac{(L_h(k))^{\alpha}}{aK^{\alpha}(1-\alpha)\gamma^k}\,\Delta Q(k) \tag{4}$$

where $\Delta Q(k) = Q(k) - Q_h(k)$. The reference trajectories $Q_h(k)$ and $L_h(k)$ can be be found in Figs. 1 and 7 respectively. Equation (4) is applicable only for fairly small changes in output Q.

In order to compute the change in aggregate wages, we also need to compute the change in wage and salary employment. This is done with a simple model of the form

$$\Delta L^w(k) = \Delta L(k) \tag{5}$$

i.e. only wage earners will be fired (or hired). We also tried a model of the form

$$\Delta L^w(k) = \Delta L_h{}^w(k)\,\frac{\Delta L(k)}{L_h(k)}$$

but it turned out to be of little significance which model was employed since the majority of the labour force are wage earners.

Capital, assets and liabilities

The increase in the fixed capital stock is equal to net fixed capital investments. Increases in government and private wealth are computed from the income distribution. Changes in net foreign debt will arise primarily from the foreign trade balance, net interest payments to foreign countries, net transfers, and changes in the foreign exchange rates.

Distributive shares

The distributive shares of the factor income consist of wage and non-wage components. The wage components are aggregate gross wages (and salaries), and employers' contribution to social security schemes. Non-wage income components are income of self-employed in agriculture, forestry, and fishing, income from dwellings, net income of foreigners from investments, etc. The components of the factor income are approximately the same as those used in the National Accounts.

Miscellaneous variables

This part of the model includes equations for direct and indirect taxes, transfer payments, subsidies, net taxes, i.e. gross taxes less transfer payments and subsidies, and a number of identities required to complete the structure. Only brief mention will be made of this part of the model, and only a very few of the most important equations will be shown.

The most important variables of the models are defined in Appendix A. State variables are denoted by $x_i(k)$, $i=1, \ldots, 31$, while non-state variables are denoted by $z_j(k)$, $j=1, \ldots, 40$. Exogenous variables are denoted by $\alpha_l(k)$, $l=1, \ldots, 13$. The vectors which consist of these variables are denoted by $\boldsymbol{x}(k)$, $\boldsymbol{z}(k)$, and $\boldsymbol{\alpha}(k)$ respectively.

Some of the miscellaneous variables are subject to government control, e.g. central government and municipal income taxes, subsidies, indirect taxes, and so on. However, the government does not directly determine the magnitude of these variables; rather, it determines central government and municipal tax rates, subsidy rates, and so on, which then determine how much tax revenues and subsidies are generated from gross incomes. For example, municipal taxes on aggregate wage income are modelled as

$$z_{37}(k)=u_{37}{}^1(z_3(k)+u_{37}{}^2) \tag{6}$$

where $u_{37}{}^1$ is the aggregate municipal tax rate while $u_{37}{}^2$ expresses, among other things, the fact that part of the income is exempted from taxation. The two constants $u_{37}{}^1$ and $u_{37}{}^2$ are the real policy instruments or control variables in this context. Of course, both $u_{37}{}^1$ and $u_{37}{}^2$ are time-varying since the government may change both tax rates and income deduction allowances. Control systems where the control actions enter multiplicatively as in eqn. (6) are usually termed bilinear or, in more general cases, variable structure systems (see Ruberti and Mohler 1975). It should be noted, however, that if we consider taxes, subsidies, etc. to be the policy instruments, instead of tax rates, subsidy rates, etc., then $z_{37}(k)$ is a control variable, and eqn. (6) is simply a specification of the controller. Both views may be appropriate in a control theory context.

The most important equations of the model are shown in Appendix B. The macrostructure of the model is of the form

$$\boldsymbol{x}(k)=\boldsymbol{\phi}(\boldsymbol{x}(k-1), \boldsymbol{x}(k), \boldsymbol{z}(k), \boldsymbol{\alpha}(k))+\boldsymbol{v}(k), \tag{7}$$

$$\boldsymbol{z}(k)=\boldsymbol{\psi}(\boldsymbol{x}(k), \boldsymbol{z}(k), \boldsymbol{u}(k), \boldsymbol{\alpha}(k)), \tag{8}$$

where $\boldsymbol{u}(k)$ denotes the control vector, i.e. the vector of policy instruments. $\boldsymbol{v}(k)$ denotes a vector of disturbance terms, i.e. the process noise vector.

The control vector $\mathbf{u}(k)$ is generally assumed to be a function of $x(k-1)$, $z(k-1)$, and, possibly, time, viz.

$$\mathbf{u}(k)=g(x(k-1), z(k-1), k-1) \tag{9}$$

since, say, tax rates for year k usually are determined in year $k-1$. We shall, however, not pursue this feedback structure any further, but simply employ open-loop control in our experiments.

A very common hypothesis in econometrics is to assume that observations are perfect, i.e. variables are observed with no uncertainty. However, it is a well-known fact that the National Accounts may contain large uncertainties. Therefore, a more realistic model of the observations may be of the form

$$y(k)=Dx(k)+w(k), \tag{10}$$

where $y(k)$ is the observation vector while $w(k)$ is the observation uncertainty or the observation noise.

The total macrostructure of the model consists of eqns. (7)–(10). It should be noted that eqns. (7)–(9) constitute an implicit non-linear set of difference equations. This type of model, usually referred to as a model in structural form, is somewhat different from the models used in engineering, which almost exclusively are assumed to be explicit, i.e. the next state $x(k)$ does not appear on the right-hand sides of eqns. (7) and (8).

3. Simulation experiments

As mentioned in § 1, the model was primarily designed as a simulation instrument to perform quantitative analyses and policy studies of the behaviour of the economy during the period 1968 through 1977, for which the equations were fitted. All experiments were designed such that the deviations from the simulated historical paths become fairly small. The results will therefore give an indication of the magnitudes of the sensitivities in the model.

In order to test the quantitative performance of the model, a simulation of the historical activity during the years 1969 through 1977 was carried out using the historical 1968 data as initial values, and with all policy instruments equal to their actual or estimated values. The root mean square errors (RMSE) of simulated nominal and real GNP, major components of nominal GNP, and the GNP implicit price deflator are listed in the table.

Variable	RMSE
GNP (nominal)	2·26
GNP (real)	2·79
Personal consumption expenditures	1·56
Gross fixed investments	1·87
Exports	0·99
Imports	2·02
GNP implicit price deflator	0·0160

Root mean square errors of simulated historical paths.

Compared with many of the large-scale models of the U.S. economy (see Fromm and Klein 1973), and taking the long simulation interval into account, the results are believed to be quite fair.

The results from this historical simulation experiment are shown in Figs. 1–12. They are referred to as the simulated historical paths or trajectories. In order to see the impact of various government policies, we shall have to compare the results with these paths, not with the real historical paths which are somewhat different. The price expectations were, in this and all other experiments, kept close very to simulated prices, i.e. we consider people's price anticipations to be practically correct.

ALL VARIABLES ARE GIVEN IN BILLIONS OF NKR EXCEPT
DEFLATOR AND MAN-YEARS WORKED

SIMULATED HISTORICAL ————
REDUCED GOVERNMENT CONSUMPTION EXPENDITURES . . . . . .
REDUCED CENTRAL GOVERNMENT INCOME TAXES - - - - - -

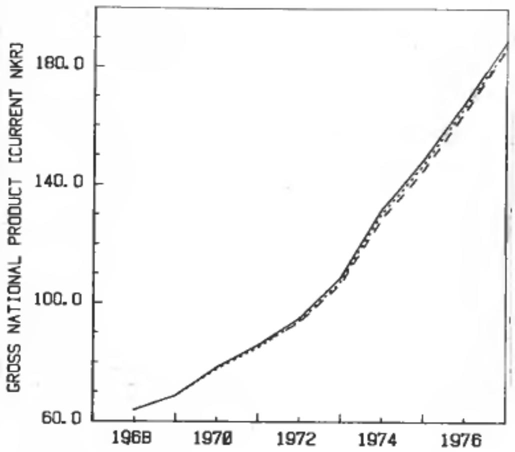
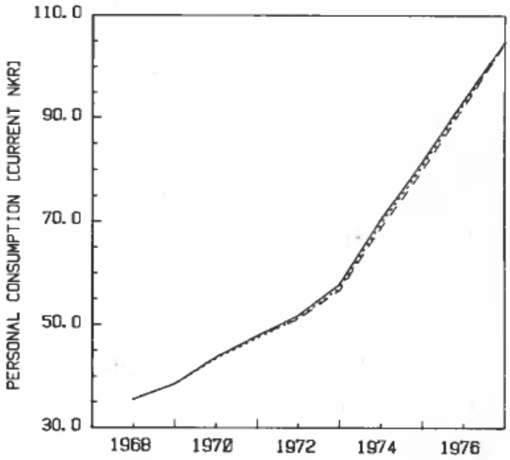


Figure 1.



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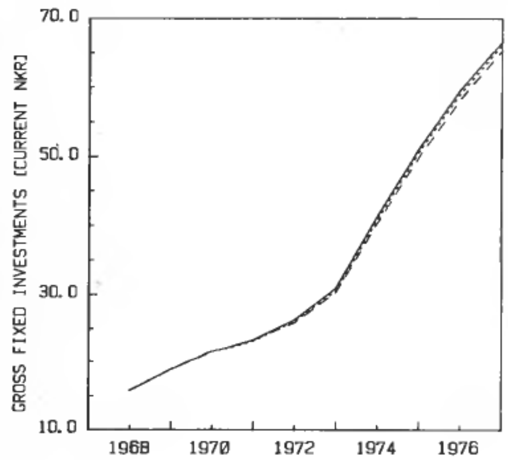

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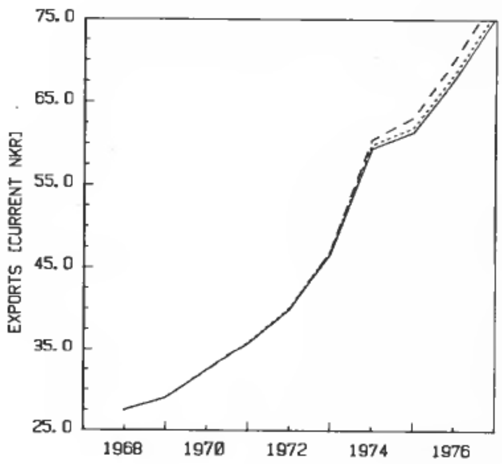

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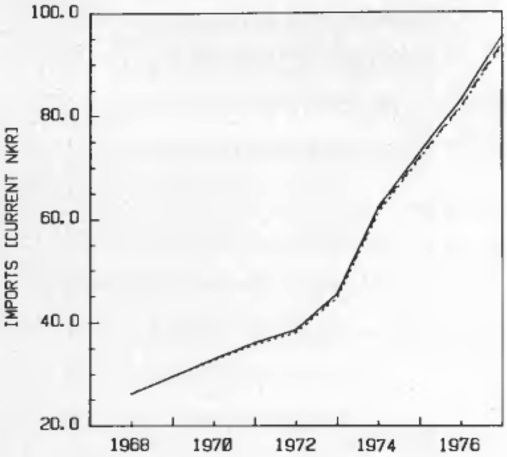

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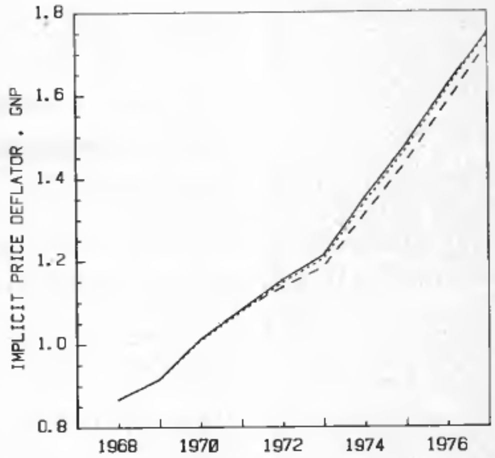

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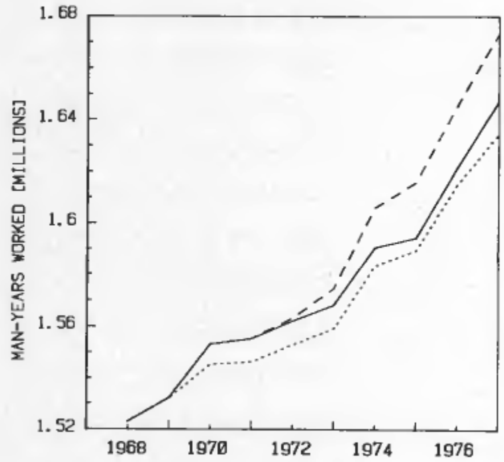

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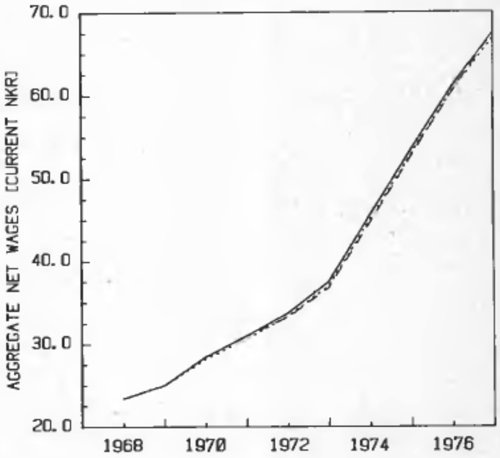

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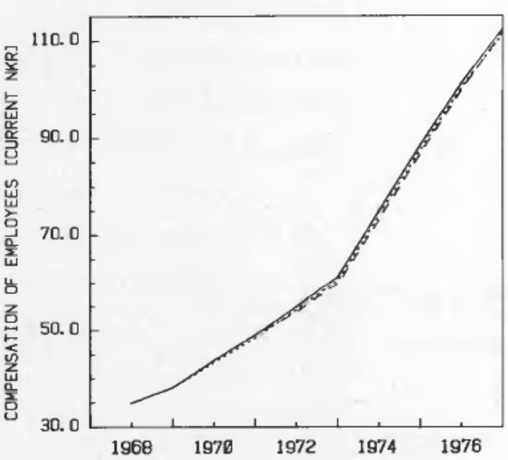

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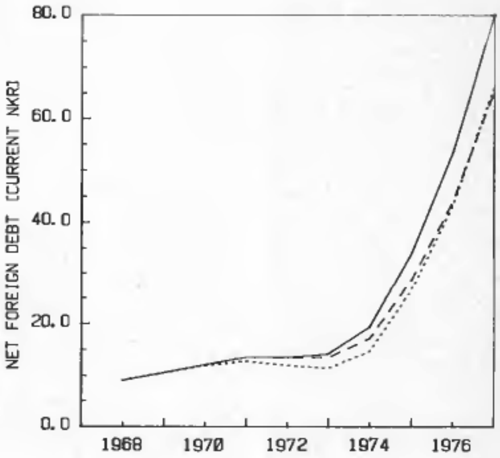

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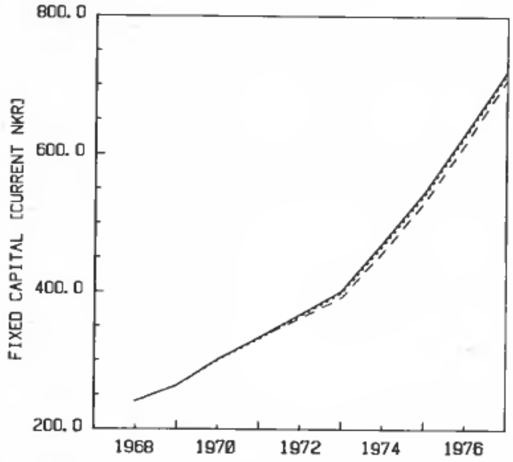

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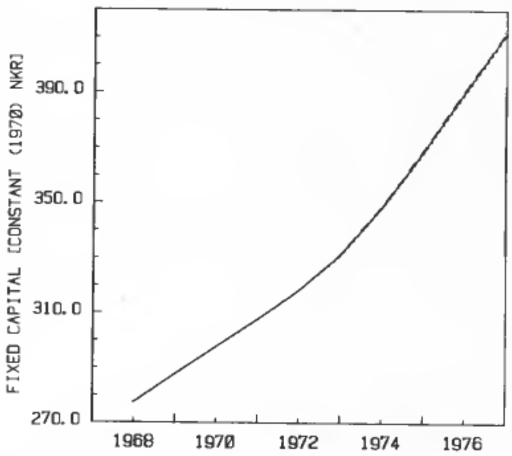

Figure 12.

Our first policy experiment was to reduce government consumption expenditures, starting with a 3% reduction in 1970 and increasing linearly to 10% in 1977. Since this will cause prices to drop, the price expectations were lowered accordingly in order to make the experiment comparable with the simulated historical experiment. The results from this are shown in Figs. 1–12. As expected, prices, GNP, personal consumption expenditures, and imports are all found to be lower, whereas exports have increased slightly (export prices are kept unchanged so that the increase stems from a larger supply). Net foreign debt has been reduced by nearly 14 billion Nkr in 1977, approximately 17% of the net debt at the end of 1977. The decrease in man–years worked is significant, 8000 in 1970, 10000 in 1972 and 13000 in 1977. This would have meant a considerable and probably unacceptable rate of unemployment compared with the relatively slight improvements in prices and foreign debt.

Now, before we go to our next experiment, let us make some comments on what impact income taxes usually are considered to have in macroeconomics. The classical and widely accepted view is that income taxes are, all other things equal, deflationary (see Branson 1972, pp. 148–166), since increased income taxes will lower aggregate demand and thus exert downward pressure on prices as long as government expenditures are kept unchanged. This effect, frequently called the income effect, is the one generally stressed in the textbooks and in discussions of the multiplier. While lowering aggregate demand, increases in the income tax rate will simultaneously raise costs, and this will put upward pressure on prices. For example, when income taxes rise, workers will demand a higher before-tax wage in order to keep their net wages unaffected. This effect, which may be dubbed the substitutional effect, is typically de-emphasized, without apparent theoretical or empirical reasons.

Blinder (1972) carried out an interesting examination, in the context of some highly simplified aggregative models, of the conditions under which one effect might dominate the other. His conclusion was that for the U.S. economy, income tax rate increases are very likely to have their desired deflationary effects. Blinder asserts, among others, the following conditions are needed to favour the tax increase having its desired deflationary effect: (1) a high income tax multiplier, (2) a low share for labour in total output, and (3) a low tax rate.

Now, let us see what conclusions can be made about Norway in Blinder's context. First, the income-tax multiplier is considerably lower in Norway since imports are much higher compared to GNP. This will make the income effects considerably smaller than in the U.S. Secondly, the shares for labour in total output are approximately the same in Norway and the U.S., approximately 0·7, i.e. rather high. This will make the substitution effects important. Thirdly, the tax rate in Norway is considerably higher, which also will tend to reverse the conclusion. Fourthly, imports and exports were neglected by Blinder. Since imports amount to nearly 50% of GNP in Norway in recent years, the downward push in prices caused by lower demand will be considerably less. Finally, increased taxes have been followed by increased government expenditures so that the income effects have been further offset. All this seems to indicate that Blinder's conclusion may have to be reversed for the Norwegian economy, i.e. increases in income taxes are inflationary. Our next experiment will suggest an answer.

In our last experiment, central government income taxes were reduced by 5% from 1972 through 1977. In order to offset the increased demand and also allow the Government to balance its budget somewhat, government consumption expenditures were decreased, so that the man–years worked did not exceed the simulated historical path with more than approximately 25000 in any year. The reduction in government consumption expenditures ranged from approximately 0·2% in 1972 to 10% in 1977. Price expectations were changed in accordance with the lower prices. The results from this experiment are shown in Figs. 1–12 for direct comparison with the other experiments. As we had anticipated, prices have dropped significantly as can be seen from Fig. 6. The nominal value of GNP has decreased slightly, but real GNP has increased as can be seen from Fig. 7 which shows that the man–years worked has increased. Therefore, in spite of higher demand and lower unemployment, prices have dropped significantly as opposed to the general belief.

Personal consumption expenditures have increased somewhat in real terms, while net fixed investments are virtually unchanged as can be seen from Fig. 12 which shows the fixed capital stock at constant (1970) prices.

The change in net foreign debt is significant and amounts to approximately 15 billion Nkr in 1977 (see Fig. 10). This is quite remarkable in lieu of the higher activity level of the economy. The change is primarily caused by an increase in exports (Fig. 4) and a drop in imports (Fig. 5). The reduction of net foreign debt is fairly moderate in the first few years after 1972, but then becomes more and more significant because of the accumulative effects. Whether the 15 billion Nkr reduction is a fair estimate of what actually would have happened is of course an interesting point. However, the significant part of the result is that the foreign debt seems to be very sensitive to changes in the income tax rates in the long run. It does not matter so much whether the reduction would actually have been 10 instead of 15 billion Nkr. The experiment shows that even a moderate reduction of 5% in central government income taxes will produce a significant drop in prices and net foreign debt, with no increase in unemployment.

The last experiment is in our opinion by far the most interesting. First, it indicates that income tax reductions may be a very efficient way to combat inflation as opposed to what may previously have been believed and what the textbooks say. Secondly, there seem to exist very high long-term sensitivities which we believe are not taken into account in economic planning and forecasting today. These sensitivities may make the risks associated with certain government policies extremely high.

4. Conclusion

The model presented in this paper has in our opinion indicated some interesting answers to questions which have been heatedly debated in Norway. First, it shows that a reduction in government consumption expenditures alone would have lowered prices and the net foreign debt somewhat, but only at the cost of a significant increase in unemployment. Second, a reduced income tax rate combined with a reduction of government expenditures could have strengthened the economy, e.g. lowered prices and the net foreign debt, without any increase in unemployment. In fact, a reduction in central government income taxes would alone have created a significant improvement, but a reduction in government consumption expenditures had to be introduced because of the limited labour force. Although a rather crude model, we still feel the experimental results to be at least qualitatively correct.

Further work is required in several areas in order to improve the present model. These areas include more refined models for personal consumption expenditures, investments, exports, and imports. Furthermore, introduction of a monetary model, more policy variables, and introduction of several production sectors seem to be important refinements. We believe that such a model can be used both for simulation, forecasting, and planning purposes, and that it may be an important supplement to the models which currently are used for planning and forecasting in Norway.

Appendix A

Definition of some important variables

Variables followed by * are lagged variables being used because of the state representation. All variables are given in nominal terms (current Nkr) unless otherwise stated.

A.1. *State variables*

x_1 personal consumption expenditures
x_2 gross fixed investments
x_3 inventory changes
x_4 implicit price deflator, GNP
x_6 net wages (aggregate)
x_7 exports
x_8 fixed capital
x_{10} net foreign debt
x_{12}^* man–years worked
x_{13}^* implicit price deflator, exports
x_{14}^* implicit price deflator, GNP
x_{15}^* price expectations
x_{16}^* gross national product
x_{17}^* effective foreign exchange rate
x_{18} man–years worked
x_{19}^* foreign wage rate
x_{20} inventories
x_{21}^* implicit price deflator, inventories
x_{22} man–years worked by employees
x_{23} depreciation
x_{29}^* implicit price deflator, imports

x_{30} imports
x_{31} implicit price deflator, government consumption expenditures

A.2. *Non-state variables*

z_1 net national product
z_2 factor income
z_3 gross wages (aggregate)
z_4 private disposable income (of Norwegians)
z_7 foreign trade deficit
z_{13} net tax revenues
z_{15} gross non-wage income
z_{16} compensation of employees, i.e., gross wages, plus employers' contributions to social security schemes
z_{20} national wealth
z_{21} gross investments in fixed capital and inventories
z_{24} gross wage rate
z_{26} gross national product
z_{27} employers' contributions to social security schemes
z_{28} indirect taxes less investment taxes
z_{29} social security premiums
z_{30} municipal taxes
z_{31} central government taxes
z_{32} transfer payments
z_{33} subsidies
z_{34} investment taxes
z_{36} social security premiums paid by employees
z_{37} municipal taxes paid by employees
z_{38} central government taxes paid by employees

A.3. *Exogenous variables*

α_1 implicit price deflator, imports
α_2 a reference trajectory for man–years worked by employees
α_3 foreign wage rate
α_4 foreign production
α_5 effective foreign exchange rate
α_6 price expectations
α_7 implicit price deflator, inventories
α_8 a reference trajectory for man–years worked
α_{10} implicit price deflator. exports
α_{11} a reference trajectory for GNP
α_{12} government consumption expenditures (real)
α_{13} net investments in the crude oil production sector (real)

A.4. *Policy instruments* (*control variables*)

These include both income and excise tax rates, government consumption expenditures, subsidy rates, social security premium rates, and so on. A few of the policy instruments are given in Appendix B, § B.5. It should be noted that the parameters of these equations are the policy instruments in this context.

Appendix B

Equations of the model

Only major equations are listed. Disturbance terms are omitted. A deflated variable is denoted by $\tilde{\ }$, e.g. $\tilde{z}_{26}$

B.1. *Components of GNP*

Personal consumption expenditures:

$$x_1(k)=\beta_1{}^1\,\frac{x_4(k)}{x_4(k-1)}\,x_1(k-1)+\beta_1{}^2\,z_4(k)+\beta_1{}^3\,x_4(k) \tag{B 1}$$

Gross fixed investments:

$$x_2(k)=x_{23}(k)+x_4(k)\{[\beta_2{}^1(\tilde{z}_{13}(k-1)+\tilde{z}_{15}(k-1))+\beta_2{}^2]+[\beta_2{}^3(z_2(k-1)-x_{22}(k-1)-z_{25}(k-1))+\beta_2{}^4]\}+\alpha_{13}(k)x_4(k) \tag{B 2}$$

Inventory changes:

$$x_3(k)=\alpha_7(k)\{\beta_3{}^1(\tilde{x}_1(k)+\tilde{x}_2(k)+\tilde{x}_7(k))+\beta_3{}^2(\tilde{x}_7(k)-\tilde{x}_7(k-1))+\beta_3{}^3(\tilde{x}_{30}(k)-\tilde{x}_{30}(k-1))+\beta_3{}^4\tilde{x}_{20}(k-1)+\beta_3{}^5\} \tag{B 3}$$

Exports:

$$x_7(k)=\alpha_{10}(k)\left\{\beta_7{}^1\,\tilde{x}_7(k-1)+\beta_7{}^2\,\frac{\alpha_4(k)}{100}+\beta_7{}^3\,\frac{\alpha_{10}(k)\alpha_5(k)}{\alpha_1(k)\times 100}+\beta_7{}^4\,\frac{\alpha_{10}(k-1)}{x_4(k-1)}+\beta_7{}^5\right\} \tag{B 4}$$

Depreciation:

$$x_{23}(k)=\beta_{23}{}^1\,\frac{x_4(k)}{x_4(k-1)}\,x_8(k-1)+\beta_{23}{}^2\,x_4(k) \tag{B 5}$$

Imports:

$$x_{30}(k)=\alpha_1(k)\left\{\beta_{30}{}^1\,\tilde{x}_{30}(k-1)+\beta_{30}{}^2(\tilde{x}_1(k)+\tilde{x}_2(k)+\alpha_{12}(k))+\beta_{30}{}^3\,\frac{\alpha_1(k)}{x_4(k)}+\beta_{30}{}^4\right\} \tag{B 6}$$

Gross national product (nominal):

$$z_{26}(k)=x_1(k)+x_2(k)+x_3(k)+x_7(k)+\alpha_{12}(k)x_{31}(k)-x_{30}(k) \tag{B 7}$$

Gross national product (real):

$$\tilde{z}_{26}(k)=\frac{x_1(k)}{x_4(k)}+\frac{x_2(k)}{x_4(k)}+\frac{x_3(k)}{\alpha_7(k)}+\frac{x_7(k)}{\alpha_{10}(k)}+\alpha_{12}(k)-\frac{x_{30}(k)}{\alpha_1(k)} \tag{B 8}$$

B.2. *Prices, wages and employment*

Implicit price deflator, GNP:

$$x_4(k)=x_4(k-1)\frac{(1-\xi(k))}{(1-\xi(k-1))}\left[1+\beta_4{}^1\frac{\gamma(k)-\gamma(k-1)}{z_{26}(k)}+\beta_4{}^2\left(\frac{\alpha_1(k)}{x_{29}(k)}-1\right)+\beta_4{}^3\left(\frac{z_{16}(k)x_{18}(k)}{z_{26}(k)x_{22}(k)}-\beta_4{}^4\right)+\beta_4{}^5\right] \quad \text{(B 9)}$$

where

$$\xi(k)=1-\frac{z_{28}(k)+z_{34}(k)-z_{33}(k)}{z_{26}(k)}, \quad \gamma(k)=\frac{z_{16}(k)}{x_{22}(k)}$$

Implicit price deflator, government consumption expenditures:

$$x_{31}(k)=\beta_{31}{}^1\, x_{31}(k-1)+\beta_{31}{}^2\, x_4(k)+\beta_{31}{}^3\frac{z_{16}(k)}{x_{22}(k)}+\beta_{31}{}^4 \quad \text{(B 10)}$$

Net wages:

$$x_6(k)=\left(\frac{\rho\alpha_6(k)+(1-\rho)x_4(k)}{x_4(k-1)}\right)\frac{x_{22}(k)}{x_{22}(k-1)}x_6(k-1)+\left(\frac{x_4(k-1)}{\rho x_{15}(k-1)+(1-\rho)x_4(k-1)}-1\right)\frac{x_{22}(k)}{x_{22}(k-1)}x_6(k-1)+\beta_6{}^1\, x_{22}(k)x_4(k)\left[\frac{z_{26}(k-1)}{x_4(k-1)x_{18}(k-1)}-\frac{x_{16}(k-1)}{x_{14}(k-1)x_{12}(k-1)}\right]+\eta(k) \quad \text{(B 11)}$$

where $\eta(k)$ is a dummy variable equal to 0 except in 1974, 1975, and 1976.

Gross wages:

$$z_3(k)=x_6(k)+z_{36}(k)+z_{37}(k)+z_{38}(k) \quad \text{(B 12)}$$

Compensation of employees:

$$z_{16}(k)=z_3(k)+z_{27}(k) \quad \text{(B 13)}$$

Man–years worked:

$$x_{18}(k)=\alpha_8(k)+\Delta x_{18}(k) \quad \text{(B 14)}$$

where

$$\Delta x_{18}(k)=(\tilde{z}_{26}(k)-\alpha_{11}(k))\frac{(\alpha_8(k))^{\alpha}}{\beta_{18}{}^1(\tilde{x}_8(k-1))^{\alpha}(1-\alpha)\gamma^k}$$

Man–years worked by employees:

$$x_{22}(k)=\alpha_2(k)+g(\Delta x_{18}(k)) \quad \text{(B 15)}$$

B.3. *Capital, assets and liabilities*

Fixed capital:

$$x_8(k)=\frac{x_4(k)}{x_4(k-1)}x_8(k)+x_2(k)-x_{23}(k) \quad \text{(B 16)}$$

Net foreign debt:

$$x_{10}(k)=\frac{x_{17}(k-1)}{\alpha_5(k)}\,x_{10}(k-1)-x_7(k)+x_{30}(k)+x_{27}(k) \qquad \text{(B 17)}$$

Inventories:

$$x_{20}(k)=\frac{\alpha_7(k)}{x_{21}(k-1)}\,x_{20}(k-1)+x_3(k) \qquad \text{(B 18)}$$

National wealth:

$$z_{20}(k)=x_8(k)+x_{20}(k)-x_{10}(k) \qquad \text{(B 19)}$$

B.4. *Distributive shares*

Factor income:

$$z_2(k)=z_{26}(k)-x_{23}(k)-z_{28}(k)-z_{34}(k)+z_{33}(k) \qquad \text{(B 20)}$$

Gross non-wage income:

$$z_{15}(k)=z_2(k)-z_{16}(k) \qquad \text{(B 21)}$$

z_{15} is then divided into the different non-wage components.

B.5. *Miscellaneous variables*

We present a few important equations in this part of the model.

Employers' contributions to social security schemes:

$$z_{27}(k)=u_{27}{}^1\, z_3(k) \qquad \text{(B 22)}$$

Indirect taxes except investment taxes:

$$z_{28}(k)=u_{28}{}^1(z_{26}(k)+u_{28}{}^2) \qquad \text{(B 23)}$$

Subsidies:

$$z_{33}(k)=u_{33}{}^1(z_{26}(k)+u_{33}{}^2) \qquad \text{(B 24)}$$

Investment taxes:

$$z_{34}(k)=u_{34}{}^1(x_2(k)+u_{34}{}^2) \qquad \text{(B 25)}$$

Social security premiums paid by employees:

$$z_{36}(k)=u_{36}{}^1(z_3(k)+u_{36}{}^2) \qquad \text{(B 26)}$$

Municipal taxes paid by employees:

$$z_{37}(k)=u_{37}{}^1(z_3(k)+u_{37}{}^2) \qquad \text{(B 27)}$$

Central government taxes paid by employees:

$$z_{38}(k)=u_{38}{}^1(z_3(k)+u_{38}{}^2) \qquad \text{(B 28)}$$

Appendix C

Estimation of parameters in the model

Estimates of the unknown parameters in the model were obtained using a general computer program developed by Hertzberg (1970), and a more specialized program developed by Opdal (1976). Both programs are essentially based on maximum likelihood estimation of parameters in models with a fixed structure. The first one allows non-linear models, but observations are assumed to be perfect, whereas the latter only allows linear single output models, but observations may be uncertain. Using different hypotheses, one can employ several well-known methods with both programs, e.g. ordinary least squares (OLS), weighted least squares, maximum likelihood, etc.

Although a very common hypothesis in econometrics, it is generally acknowledged that observations in economics may be very uncertain. This may create biased estimates when ordinary econometric methods are employed. Another problem is created by the fact that the model is implicit, i.e. $x(k)$ and $z(k)$, which correlated with $v(k)$, both appear on the right-hand side of eqn. (7). If some of the right-hand variables are correlated with $v(k)$, the parameter estimates will generally become biased even if the observations are perfect. There exist methods in econometrics which are designed to resolve such problems, e.g. two-stage least squares (TSLS), but they are all based on the assumption that observations are perfect.

The number of known parameters in the model amounts to approximately 75 which is considered to be quite large compared to typical engineering problems. Because time was limited, it was therefore decided to employ ordinary least squares techniques, either by fitting single equations or groups of strongly coupled equations. As pointed out previously, this method will produce biased estimates even if observations are perfect. However, the superiority of more sophisticated methods, e.g. TSLS, to OLS is not unambiguous when model errors are taken into account. In a recent paper, Hale *et al.* (1978) point out that the relative insensitivity of OLS to model errors outweighs its disadvantage in terms of bias and mean squared error, and thus will often be preferable to TSLS. In view of the fact that (1) the model is very crude, and (2) observations are imperfect, we feel that the use of the simple OLS method can be justified.

The only way to resolve the problem with imperfect observations is to employ Kalman filtering or related techniques. The use of these techniques in non-linear econometric models has recently been outlined by one of the authors (Henriksen 1979).

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