MODELING, IDENTIFICATION AND CONTROL, 1980, VOL. 1, NO. 4, 247–258
doi: 10.4173/mic.1980.4.4

Multivariable control in aluminium reduction cells

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Keywords: *aluminium reduction cells, multivariable control, nonlinear model, stochastic estimation algorithms.*

The paper presents a multivariable control system for an aluminium reduction cell. The system is based on a dynamic model for the energy and mass balance. This model gives an optimal stationary state (or behavior) of the cell and is the basis of a reduced model which is used in calculation of the feedback control.

The process has certain peculiarities which must be taken into account when dealing with process control. It has considerable non-linearities, and if you want to use a linear reduced model, there is no real stationary point where it is possible to calculate this model. This is due to the fact that the process has continuous production of aluminium and consumption of raw materials, while the tapping and the supply are batch processes. The measurements that are available are few and, therefore, much work has been expended on making the model as accurate as possible. Taking into consideration the observability and controllability conditions, an alternating filter based on a Robbins Monro algorithm is used, which can only adjust some of the state variables. The state variables to be adjusted are dependent on the disturbances and the controls. The control problem is solved as an ordinary linear problem for the anode adjustment. For the supply of raw materials the feedback control is calculated in a more direct way, taking into account that it is a batch process, and that we want to keep control with the mass of undissolved alumina. The paper also contains some experimental data for the control system.

1. Introduction

The main goal of this project has been to construct a multivariable control system to obtain as good operation of the pots as possible. The consequences of good operation is good economy, less uncomfortable jobs and smaller gass polution. The project has been a joint project between A/S Årdal og Sunndal Verk (ÅSV) and Institutt for Energiteknikk (IFE). ÅSV is the biggest aluminum producer in Norway, and have for several years used computer models in construction and parameter studies in for example aluminum reduction cells.

Today, the computer control system is divided into several parts which operate almost independently of each other. One part takes care of the measurement of the ohmic resistance through the pot, and controls it by aiming at a target value set by the foreman. Supply of alumina is done through a special schedule based on anode effect voltage and frequence. Events like tapping, changing of anode blocks and AlF_3-supply are ordered by the foreman on the basis of a time-dependent routine. This computer system works very well. It was installed in 1968 and 2–3 years after, almost all the pots were atuomatically controlled. The problem is, however, that sometimes the pots become partly unstable or the pots behaviour are not optimal. The reason for this is that the conventional system does not control the bath temperature, the thickness of the side freeze or the mass of undissolved alumina. The question is then, is it

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possible to control the entire mass- and energy balance of the pots using a mathematical model and multivariable control theory? If the answer is yes, what does this system require of measurements and control equipment, and how will it affect the workers who operate the pots? This paper will mainly deal with the first question, while the total project also deals with the other questions. Concerning the multivariable control system, the project objectives may be summarized as follows:

(1) Low energy consumption and maximum current efficiency. (This is dependent on the concentration of the dissolved alumina and fluoride in the bath, the temperatures in the pot and the interpolar distance.)
(2) Safe operation of the pot. (We want for example to get good control of the side freeze and undissolved alumina to prevent leakages or unstable working conditions.)
(3) Managing the interuptions to desirable points of time. (We want for example anode effect to happen in daytime.)
(4) Use the available measurements and knowledge of the pots conditions to get reliable and good information for state estimation.

2. Description of the process

The production of aluminium takes place in electrolytic cells (pots) where alumina is dissolved in molten cryolite. The alumina is reduced electrolytically to Al at temperatures of 950–970°C. A cross-section of a pot is shown in Fig. 1.

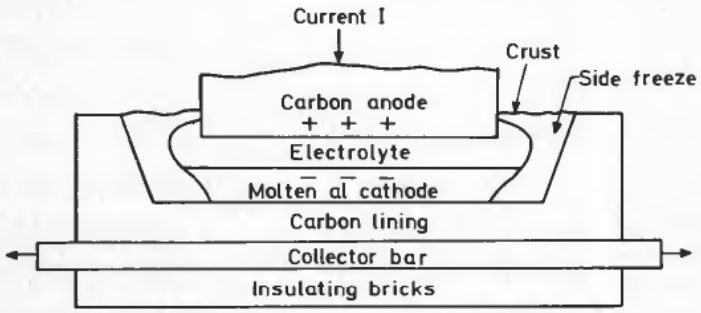

Figure 1. Cross section an aluminium reduction cell.

The most important factors concerning energy balance of the pot, are the amperage through it, the ohmic resistance between the anode and the cathode, the thickness of the side freeze, and the energy used in dissolvation and reduction of alumina. The process has considerable nonlinearities and the pot is never completely stationary because the production of aluminium and the consumption of raw materials are continuous, while tapping of aluminium and supplying of raw materials takes place at discrete points of time. When certain controls are performed on the pot it involves great disturbances, for example when the crust is broken down into the pot during the supply of raw materials.

However, the pot also has some good properties. The measurements of the voltage and the amperage are reliable, and it is possible to get a good estimation of the ohmic part of the voltage. When the ohmic effect produced in the pot is known,

the variation of the side freeze gives a negative feedback on the temperatures in the pot, and the energy balance is therefore a stable process. When the cell is operated without an adequate supply of alumina, the so called 'anode effect' occurs. This shows itself as a high voltage, and this effect gives you information about the mass balance in the cell. It is also possible to get some information from the response of certain controls you perform on the cell. Further, the production of aluminum and the consumption of raw materials is given very accurately by the current through the pot, and if you get good control (estimate) of the side freeze, the undissolved alumina, and the temperatures, you can calculate the concentration of raw materials in the bath. These concentrations are very important for the current efficiency and the energy consumption.

3. The model of the energy and mass balance

The pot is divided into a finite number of components and the state of one such component is described by its mean state. This leads to a model consisting of concentrated parameters:

$$\dot{x}=f(x, u, t) \tag{1}$$

$$y=g(x, u, t) \tag{2}$$

where

x is a n-dimensional state vector
u is a r-dimensional control vector
y is a m-dimensional measurement vector
t is the time.

The way the pot is divided up is shown in Fig. 2. Today the model has 26 different state variables and 9 different controls. To integrate these 26 nonlinear equations, fifth order Runge-Merson formulae are used. A more detailed description of the model is given in Ek and Fladmark (1973). Since the model is nonlinear and perhaps some of the states are superfluous in an estimator working on-line with the process, we want to make a linearization of the model based on ordinary perturbation techniques. In order to do that we want to define a stationary solution, despite the fact the process itself has no stationarity, and to make the perturbations around a nominal trajectory. The stationary solution will be the initial state of this trajectory. If the initial state is different from this, we will not get this desired trajectory.

Since we want the pot to be near an optimal state, the stationary solution and thereby the initial state of the trajectory should be optimal. The model (1) calculates among other things the current efficiency and energy consumption for a stationary solution. One procedure is to specify the (optimal) concentrations of raw materials and the thickness of the side freeze, and let the model calculate a corresponding distance between the anode and cathode, and the temperatures inside the pot. The stationary solution is defined by letting all the time derivatives of the dynamic variable be equal zero:

$$f(x, u, t)=0 \tag{3}$$

For the masses this means that we must have a continuous tapping of aluminum and adding of raw materials. The solution we get will then represent the mean states of the pot over a long period with constant, or periodically repeated, operating conditions. To solve (2) we use a Newton-Raphson method with a special treatment of singularities (Rasmussen 1970).

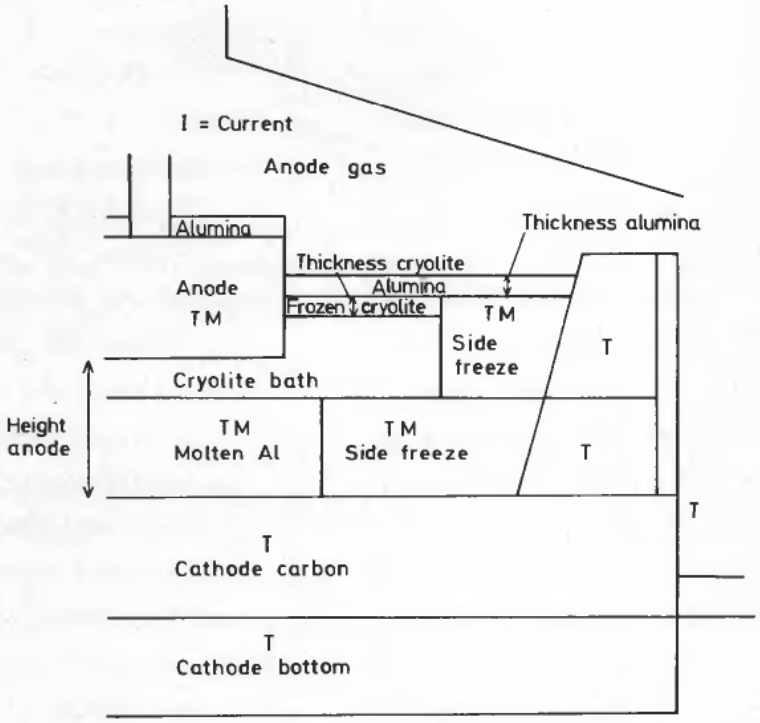

Figure 2. Components of the model. T=temperature of the component. M=mass of the model.

4. The reduced linear model

As mentioned above, the optimal stationary solution is the initial state in calculating the reduced model. We use ordinary perturbation techniques to describe how 'irregularities' are transmitted in the pot (Saksvikrønning, Gran and Vee 1976). The procedure is as follows. We simulate the model one sampling period from the stationary solution and get new states for the nominal trajectory at the end of the sampling period. To get a model describing how the 'irregularities' from this nominal trajectory are being transmitted in the pot, we go back to the initial states of the nominal trajectory (stationary solution), perturbate every single state one at a time and simulate the model the same sampling period. The states at the end of the period this time, will not be the same as the states of the nominal trajectory since we have initially perturbated one of them. The difference between these two set of states are used to calculate one column in the transition matrix Φ_1. The matrix Φ_1 will describe the transition:

$$\boldsymbol{x}_1(k+1)-\boldsymbol{a}(k+1)=\Phi_1(\boldsymbol{x}_1(k)-\boldsymbol{a}(k)) \tag{4}$$

where

$\boldsymbol{a}(k)$=the state vector of the nominal trajectory at sampling point k

$\boldsymbol{a}(k+1)$=the state vector of the nominal trajectory at sampling point $k+1$

$\boldsymbol{x}_1(k)$=the actual state vector at sampling point k

$\boldsymbol{x}_1(k+1)$=the actual state vector at sampling point $k+1$

We can modify eqn. (4) to a normal form:

$$x(k+1)=\Phi x(k) \tag{5}$$

where

$$x(k)=\begin{bmatrix} x_1(k) \\ \cdots\cdots \\ 1 \end{bmatrix} \tag{6}$$

$$\Phi=\left[\begin{array}{c|c} \Phi_1 & b \\ \hline 0\ldots\ldots 0 & 1 \end{array}\right] \tag{7}$$

and

$$b=a(k+1)-\Phi_1 a(k) \tag{8}$$

From this, we see that the vector b contains the nominal trajectory and a correction due to the fact that the state vector $x(k)$ is the actual state and not a difference from the nominal trajectory.

By fairly the same method as calculating the transition matrix Φ_1, we can calculate the transmission matrix Δ and the measurement matrix D. The model is then described by the eqn. (9), (10).

$$x(k+1)=\Phi \cdot x(k)+\Delta \cdot u(k) \tag{9}$$

$$y(k+1)=D \cdot x(k+1)+E \cdot u(k) \tag{10}$$

$x(k)$, $u(k)$, and $y(k)$ are the vectors defined below eqn. (1, 2) at sampling no. k. Φ is a nxn transition matrix. Δ is a nxr transmission matrix. D is a mxn measurement matrix.

Also, the reduction of the number of state variables is produced during this linearization procedure. We only perturbate and calculate the transition matrix Φ_1, for those states which:

(1) we want to keep in the model for control purposes,

(2) must be included to preserve the dynamics of the pot.

On the basis of these factors we have concluded that the estimation model must have the variables stated in Table 1.

In the linear model eqns. (9) and (10) we have to include 2 nonlinearities which are essential in our process.

Δ contains one nonlinear element describing the instant dissolving of alumina in the bath. D contains one nonlinear element describing how the resistance in pot is a function of the concentration of dissolved alumina in the bath. We have to use a matrix E to describe how the current (a control variable) influences the resistance. Consequently, to be correct, the linear model contains 2 important nonlinearities which are necessary to give the proper accuracy.

The nominal trajectory is given implicit in the Φ-matrix as a constant. The linear model has been compared with the nonlinear model eqn. (1, 2) in several simulation studies. The reduced linear model contains 13 state variables. Linear models based on perturbations around a nominal trajectory can give problems dealing with feedback control, if the trajectory is not described good enough. However, in this case,

State variables	Measurements	Controls
1. Mass of aluminum	1. Resistance	1. Tapping of aluminum
2. Mass of bath	2. Percentage of dissolved alumina	2. Supply of alumina by crustbreaking the side
3. Mass of dissolved alumina in the bath	3. Percentage of fluoride	3. Supply of alumina in the center of the pot (controlled by the computer or manually)
4. Mass of aluminum fluoride	4. Temperature in the bath	4. Supply of aluminum fluoride
5. Mass of the side-freeze in the bath level	5. Temperature in the carbon side blocks	5. Changing of anode carbons
6. Mass of the side-freeze in the aluminum level	6. Aluminum height	6. Anode adjustment
7. Mass of undissolved alumina in the bath	7. Bath height	7. Amperage
8. Mass of undissolved alumina in the aluminum	8. Interpolar distance	8. Anode effect (is a control to the model, not the process)
9. Volume of frozen cryolite in the aluminum	9. Thickness of the side freeze in the bath level	
10. Temperature in the bath	10. Thickness of the side freeze in the aluminum level	
11. Temperature in aluminum		
12. Height anode		
13. Constant = 1		

Table 1. State variables, measurements, and controls in the reduced linear model.

describing the nominal trajectory as a constant deviation from time k to $k+1$ appears to be accurate enough. It also depends on how one solves the feedback problem. (See also the example at the end of this paper.)

5. The multivariable control system

As a starting point in this project we used ordinary extended Kalman-filter techniques. See Saksvikrønning, Gran and Vee (1976). The measurements of the resistance and the amperage are reliable and good measurements. At an earlier stage in this project we believed strongly in measuring the temperatures in the sideblocks on-line. However, these measurements were noisy, more influenced by crustbreaking, dissolving of alumina, and other strongly local phenomena inside the pot, than the thickness of the side freeze and the mean temperatures in the bath and aluminum.

As in ordinary Kalman-filter techniques we want the model to work in parallel with the pot. This is possible by using the resistance in a Robbins-Monro stochastic approximation algorithm as described in Martin (1972). First let us look at the important features which make this possible:

1. When the ohmic effect produced in the pot is known, we have a *stable* process.
2. Some of the important variables are slowly varying and can be measured from time to time.

3. The consumption of raw materials and production of aluminum is given very accurately by the current through the pot.
4. When anode effect occurs it will give you good information of the concentration of dissolved alumina.
5. When you perform certain controls on the pot (as for example breaking the crust) you know what variables that will be affected.
6. Some of the controls give you an informative response in the resistance.
7. The mass of tapped aluminum is measured accurately.
8. The amount of supply of alumina are nearly constant.

By using only the resistance as an on-line measurement, we get of course almost open-loop prediction between the controls. This gives very strong demands on the model for how well it can simulate the process, and the testing of the model against measurement data from the process has been the most time consuming but also the most interesting part in this project. The simulation of the model against data from the process has been a powerful tool in better understanding in how the physical relationships interact in the pot.

The energy input to the model is controlled by the following Robbins-Monro algorithm (Martin 1972),

$$x'(k)=x'(k/k-1)+\frac{A}{D'}\psi(y'(k)-y'(k/k-1)) \tag{11}$$

where

$$y'(k/k-1)=\tilde{D}\boldsymbol{x}(k/k-1)+\tilde{E}\boldsymbol{u}(k) \tag{12}$$

$x'(k)$ is the corrected state variable (not a vector) at time $\hat{k}$.

$x'(k/k-1)$ is predicted state variable calculated by the model (9) at time k. It is based on measurements up to time $k-1$.

A is a gain constant in the stochastic algorithm. (For example $A=0{,}5$).

D' is the matrix element in the D-matrix (10) connecting resistance and the state variable $x'(k/k-1)$.

$y'(k)$ is the measured resistance through the pot.

$y'(k/k-1)$ is the predicted resistance calculated by the model (9, 10).

ψ is the influence function for the algorithm. See Fig. 3. It minimized the variance for noise densities which are contaminated gaussian.

$\tilde{D}$ is the line in the D-matrix (10) connecting the resistance and the state vector.

$\boldsymbol{x}(k/k-1)$ is the predicted state vector calculated in the model (9).

$\tilde{E}$ is the line in the E-matrix (10) connecting the resistance and the control vector.

$\boldsymbol{u}(k)$ is the control vector defined in (10).

The eqns. (11) and (12) are used for each time step. Only one state variable is corrected each time. The state which is corrected in this manner is dependent on the last controls. The state variables are corrected in a manner such that energy input in the model is the same as the energy input in the pot. Other measurements which

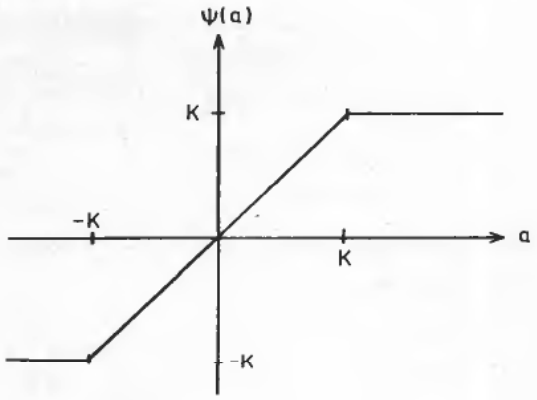

Figure 3. The influence function used in the stochastic algorithm. K is generally a function of the noise variance and the contamination factor. See (Martin 1972). (We have used $K=1\ \mu\Omega$).

are not performed on-line each time-step, can be used to correct the model manually or the model can calculate the correction factor automatically and correct the state variables that influence the actual resistance. This is for example the case for the measurements of bath height and the aluminum height.

If the model is going to behave accurately enough (behave like the pot we are going to control) the following must be satisfied.

(1) The energy input to the model must be the same as in the pot. This is taken care of by the Robbins-Monro stochastic algorithm described above.

(2) All 'mass controls' as adding alumina, fluoride, tapping of aluminum which are performed on the pot, must be given or 'estimated' accurately enough by the model.

(3) The model must be good. All important factors influencing the energy balance of the pot must be modelled. (Thickness of the side freeze, liquid temperature, dissolvation energy for the alumina and fluoride, reaction voltage, anode effect, etc. Some of these factors are not state variables but are given implicit in the model (1) and therefore also in (9).)

In this project, much work has been done in getting more information from measuring the resistance. For some pots, it is sometimes possible to predict the anode effect and thereby control the percentage of dissolved alumina. However, generally this is not possible. On the other hand it is possible to get information of the thickness of the sidefreeze by measuring the resistance combined with the anode adjustments, during and after tapping of aluminum.

The total control system (with the controller described in below) is shown in Fig. 4.

6. The control problem

We want to calculate a feedback matrix from the calculated state vector:

$$\boldsymbol{u}(k)=G(\hat{\boldsymbol{x}}(k)-\boldsymbol{x}_{\text{opt}}(k)) \tag{13}$$

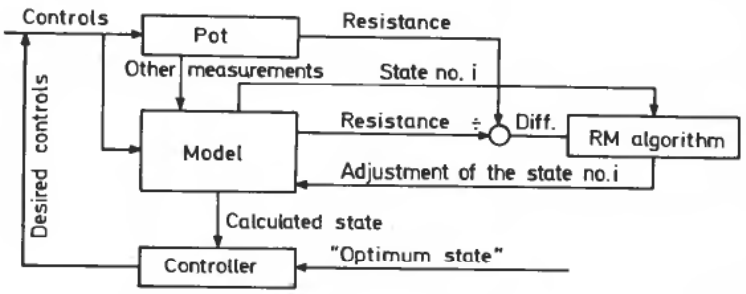

Figure 4. Multivariable control system for aluminium reduction cells.

The control matrix G describes the control of:

1. Tapping of aluminum.
2. Supply of alumina.
3. Supply of AlF_3.
4. Anode movement.

In the calculation of the G-matrix we have to keep in mind that we are dealing with a batch process and that the three first controls only can be positive. It is for example not possible to remove alumina. Further, the three first controls should not be performed too often, because they introduce great disturbances on the process. Tapping of aluminum and crustbreaking takes place on a specified time schedule for all the pots. Supply of alumina takes place by an automatic feeding (can be ordered from the system) or by breaking the crust. However, anode adjustment can be performed on-line and can be both positive and negative. We have solved the control problem by using ordinary minimum variance control as described in for example Ek and Gran (1974) for the anode movement. The performance index which is to be minimized put weights on:

1. Thickness of the side freeze.
2. Bath temperature.
3. Distance between the anode and the cathode.
4. The anode movement.

Since none of these variables are typical batch processes with a mean driving term, $x_{opt}(k)$ in (6) is a constant vector. The calculated optimal anode movement has to be above a certain limit to be performed. The elements in the G-matrix for the other controls are given in a more direct manner. Since the mass of aluminum, the mass of alumina and the mass of AlF_3 (these variables are state variables), are typical batch processes with almost constant driving terms, we have simply used the gain ($G=2$) from the according state variable to the according control variable. Some of these controls have to reach a certain limit to be performed, others are performed according to a specified schedule. However, the control of alumina supply also follows a more technical algorithm which is based on the mass of undissolved alumina and the anode effects.

7. Discussion of the total control system

In the construction of the control system we have tried to approach the problems and solve them in the most natural way. All the states in the model have for example a real physical meaning. ÅSV and IFE have had very useful discussions in this project and this has resulted in an improved understanding of the process we want to control. As a result of this the simulation models have improved constantly. Due to the state variables included in the model, both the estimation loop (RM-algorithm) and the feedback control have a PID effect. The feedback control is constructed in the most direct way. With this control, it has been very interesting to see how the pot resistance vary as a function of raw materials in the bath and as a function of time.

8. Examples

The complete system has been tested on one pot in Årdal since August 1978. Figure 5 shows a typical result of the estimation problem. Here the system controls the anode movement and the automatic supply of Al_2O_3. However, the mass of

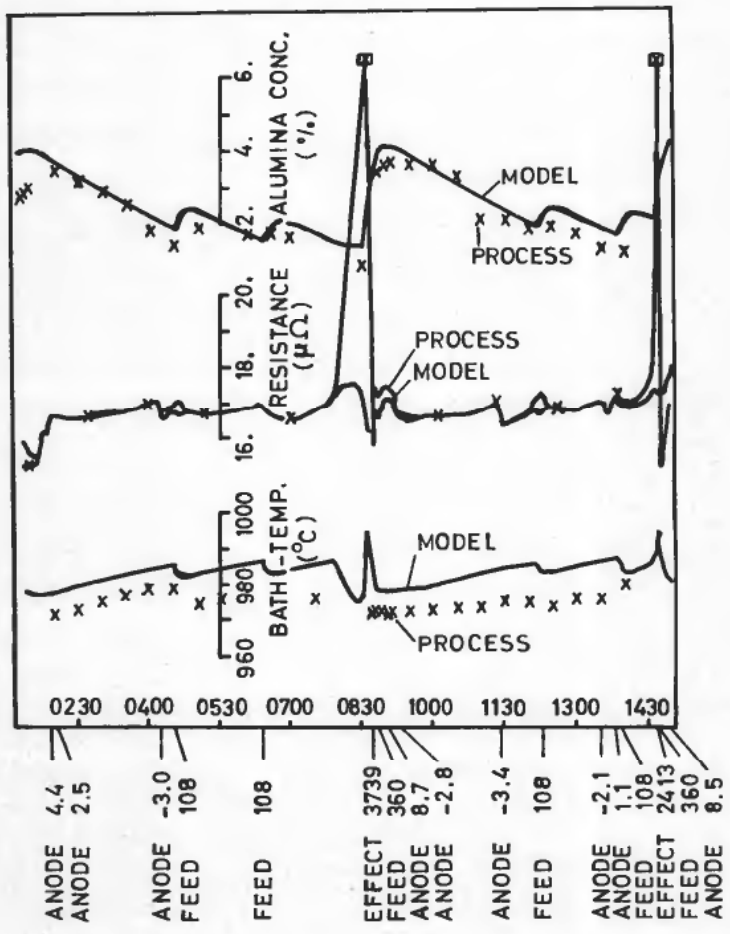

Figure 5. Estimation and control based on measuring the resistance through the pot. Anode = anode adjustment (mm). Feed = adding alumina (kg). Effect = anode effect (volt × sek).

undissolved alumina is unexpectedly high, and we get two anode effects in this period as a result of this. The bath temperature is measured in a single point in the pot, and we see that it has the same trend as the estimated mean value. We know from other measurements that we have temperature gradients in the bath as a function of the space. So the estimated bath temperature might be a good estimation. The same yields for the alumina concentration.

The next example, Fig. 6, shows a test of the linear reduced against the nonlinear model (1, 2). The nonlinear model can in this example represent the pot and both models are controlled by the feedback controller described above. This simulation shows how anode effect influences the system. In this case the adding of alumina to get rid of the anode effect is relatively small, such that anode effect and energy for dissolving the alumina represents a positive energy input. This is not the case in the first example. Notice also that the side freeze is freezing before the anode effect. The

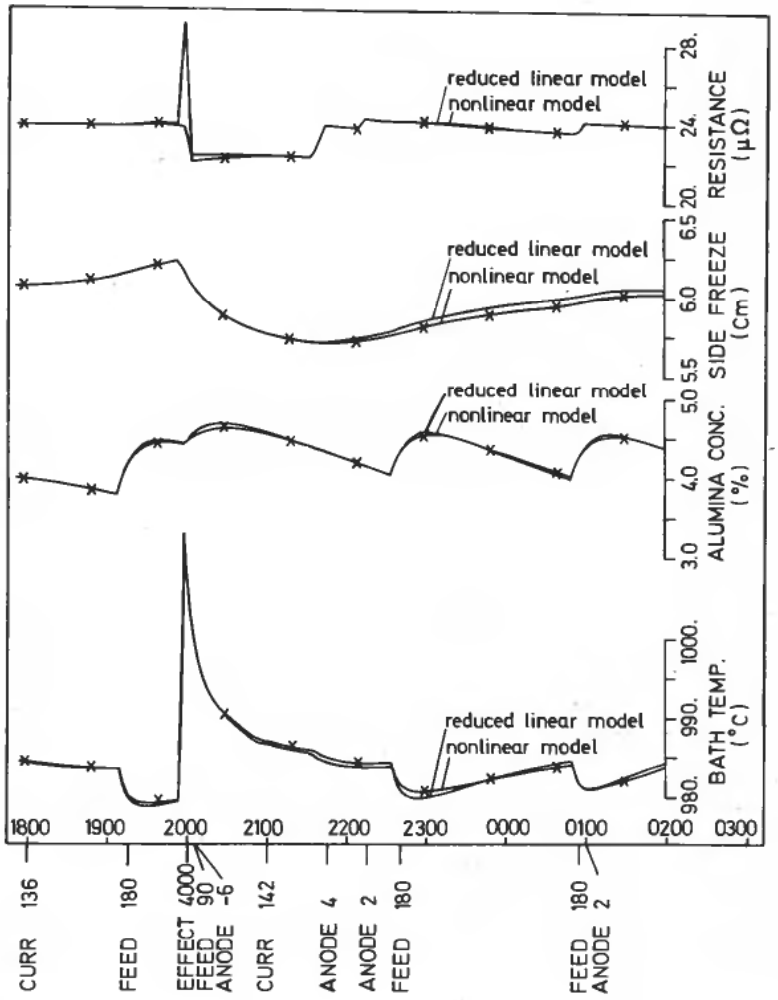

Figure 6. The linear reduced model with feedback controller against the nonlinear model (process). Anode = anode adjustment (mm). Feed = adding alumina (kg). Effect = anode effect (volt × sek). Curr = current (kA).

reason for this is that the liquids temperature for cryolite is getting higher because the alumina concentration is getting smaller. The deviation between the reduced linear model and the nonlinear model is biggest when the state is far from optimum (linearization point). This and other simulations over a longer period of time shows that deviation between the models are not growing steadily. When the models are back near the optimum state they are approaching each other after being removed from the optimum state due to anode effects, current variations, etc.

9. Conclusion

The goal in this project has been to control the entire mass and energy balance of an aluminum reduction cell. This is only possible if you get a thorough understanding of the physical relationships in the process, how they interact on each other, their importance for the energy balance, etc. These relationships must be used in a mathematical model which is not too complicated, but gives a good description of the process behaviour. The estimation and control techniques applied on this model, can then be made fairly simple as described in this project.

The entire control system is now working on-line on one pot in Årdal. We are very satisfied with the results, but it is difficult to ge a quantitative measure of the better performance. When you are concerned with the behaviour of a single pot, it will always run better than the others because it is taken more care of. Today, this pot and the estimation results are used for intensive parameter studies. This will not only be a benefit for this project but for the general research and construction of the aluminum reduction cells.

Acknowledgments

This has been a joint project between IFE and ÅSV. It has been fully paid by ÅSV and several people have worked on this project. Especially Ola Jakobsen, Tore Saksvikrønning, Kåre Vee, and Tore Solem, ÅSV, and Jon Erik Jacobsen, IFE, ought to be mentioned.

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