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Robust H_∞ Filtering for Networked Control Systems with Markovian Jumps and Packet Dropouts

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Abstract

This paper deals with the H_∞ filtering problem for uncertain networked control systems. In the study, network-induced delays, limited communication capacity due to signal quantization and packet dropout are all taken into consideration. The finite distributed delays with probability of occurrence in a random way is introduced in the network.The packet dropout is described by a Bernoulli process. The system is modeled as Markovian jumps system with partially known transition probabilities. A full-order filter is designed to estimate the system state. By linear inequality approach, a sufficient condition is derived for the resulting filtering error system to be mean square stable with a prescribed H_∞ performance level. Finally, a numerical example is given to illustrate the effectiveness and efficiency of the proposed design method.

Keywords: H_∞ filter, Networked control system, packet dropouts, Markov jump system

1 Introduction

Networked control systems (NCSs) with Markovian jumps are typical complex stochastic dynamic systems, which can describe many real world systems, and much attention have been paid on stability analysis and control synthesis of this kind of complex stochastic dynamic systems, see for example Li et al. (2013b) and the references therein. Networked control systems become an important way to study complex systems due to their low cost, simple installation, maintenance and high reliability. Communication channels can reduce the cost of cables and power, simplify the installation and maintenance of the whole systems, and increase the reliability compared to the traditional point-to-point wiring system. NCSs have many applications such as remote surgery, unmanned aerial, vehicles and communication network, etc. Now, more and more efforts have been devoted to both the stability and the control of the NCSs. On the other hand, note that some inevitable phenomena when the control signals transmitted through the communication network, sev-

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eral challenging issues will appear such as time-delay, packet dropouts, quantization and so on, which may influence system performance and have been primarily highlighted in Seiler and Sengupta (2005); Zhao et al. (2011); Li et al. (2012); Peng and Tian (2007); Kim and Kumar (2013).

In practice, due to the limited transmission capacity of the network and some devices in closed-loop systems, signals should be quantized before they are sent to the next network node. In order to get better system performance, more effects of quantization in NCSs should be taken into consideration. The quantizer can be regarded as a coder which converts the continuous signal into piecewise continuous signal taking values in a finite set, which is usually employed when the observation and control signals are sent via limited communication channel. A number of results have been reported on the quantization problems in recent years, see for example Fu and Xie (2005); Tian et al. (2007); Li et al. (2013b,a); Garcia and Antsaklis (2013) and the references therein.

In NCSs, one of the important scheduling issues to treat is the effect of the network-induced delay on the system performance. For NCSs with different scheduling protocols, the network-induced delay may be constant, time-varying, or even stochastic values. There have been lots of works concerned with the analysis and synthesis problems for NCSs with network-induced delay, see for example, Karimi (2009); Li et al. (2014), and the references therein. It should be noticed that, in the network environment, the traditional methods for deterministic time delays cannot be directly employed to deal with NCSs. New approaches are desirable to be presented to cope with the effect of network-induced delay. In the literature, Markovian chains and Bernoulli process are adopted in describing stochastic time-delay in Markov systems. Among them, the stochastic time-delay NCSs modeled as Markov chains in NCSs have received much attention, see for instance Shi and Yu (2009); Liu et al. (2005); Wang et al. (2012); Shi et al. (1999); Zhang and Boukas (2009); Zhang et al. (2008) and the references therein. Different with the above methods, in order to model a realistic complex NCSs, finite distributed time delay with a certain probability is proposed in this paper, and the stochastic time delay is an independent Bernoulli process.

Another significant issue inevitably emerged in NCSs is the possibility that the data may be lost due to the limited capacity in transit through the network. Packet dropout can be modeled as stochastic or deterministic phenomena. The main stochastic method to deal with the packet dropout is by modeling the packet dropout process as an independently and identically distributed Bernoulli process. For this reason, the state estimation over networks cannot be ignored in order to achieve better performance in applications such as remote sensing, space exploration and sensor networks. Therefore, a lot efforts have been devoted to the filtering problem, see for example Zhang and Yu (2008); Li and Shi (2014); Wu and Chen (2007); Sun et al. (2008); Wang et al. (2007); Niu et al. (2009); Sun (2012); Yashiro and Yakoh (2014) and the references therein. However, so far, the filtering problem for NCSs with mixed stochastic delays, quantization and packet dropout has not been fully investigated, which motivates us for this study.

Among the existing results of NCSs, many works have been done either on time-delay modeled as Markov chains or quantization. But in practice, network stochastic delay and quantization are both quite often. However, there has been very limited work that has taken such type of multiple network-induced phenomenon into account. To the best of the authors' knowledge, up to now, little attention has been focused on NCSs with quantization, time-delay and packet dropout modeled as Markov jump system. From both theoretical and practical points of view, we should consider the problem of robust stability and immeasurabilty of network simultaneously.

The goal of this paper is to study robust H_∞ filtering problem for uncertain NCSs with quantization, time-delays and packet dropout. Partially unknown transition probabilities of Markov chain is used to model the system. The desired filter is designed by linear matrix inequality (LMI) approach. Sufficient conditions are proposed to ensure the resulting filtering error systems to be robustly mean square stable with a given H_∞ performance level. Finally, a numerical example is provided to illustrate the effectiveness of the proposed design technique.

2 Problem Formulation

Consider the following networked control systems:

$$\begin{aligned} x(k+1) =& A(r(k))x(k) + B_w(r(k))w(k) \\ & + A_d(r(k))\sum_{i=1}^{p} \lambda\{T_m = \tau_i\}x(k-i) \\ y(k) =& C(r(k))x(k) \\ z(k) =& E(r(k))x(k) \\ x(j) =& \varphi(j),\ -\infty < j \le 0 \end{aligned} \tag{1}$$

where for $k \in \mathcal{Z}$, $x(k) \in \mathbb{R}^n$ is the state vector, $y(k) \in \mathbb{R}^m$ is the measured output, $w(k) \in \mathbb{R}^p$ is the disturbance input which belongs to $\mathcal{L}_2[0,\infty)$, $z(k) \in \mathbb{R}^q$

is the state to be estimated. $\varphi(j)$, $-\infty < j \leq 0$, are the initial conditions.

The distributed time delays have a certain probability. Each time delay T_m, $m \in \{1, 2 \ldots, p\}$ is denoted as follows:

$$\lambda\{T_m = \tau_i\} = \begin{cases} 1, & T_m = \tau_i \\ 0, & T_m \neq \tau_i \end{cases} \tag{2}$$

$$Prob\{T_m = \tau_i\} = \mathbb{E}\{\lambda\{T_m = \tau_i\}\} = \beta_i,\ i = 1, 2 \ldots, p.$$

where $E\{\cdot\}$ stands for the mathematics statistical expectation of the stochastic process. $0 \leq \beta_i \leq 1$, and $\sum_{i=1}^{p} \beta_i = 1$.

The parameter $r(k)$ represents a discrete-time homogeneous Markov chain taking values in a finite set $\mathcal{I} = \{1, 2, \cdots, N\}$ with the associated transition probability matrix $\Lambda \in \mathbb{R}^{N \times N}$, whose elements are given by $p_{ij} = Pr\{r(k+1) = j | r(k) = i\}$, where $0 \leq p_{ij} \leq 1$, $\forall i, j \in \mathcal{I}$, *and* $\sum_{j=1}^{N} p_{ij} = 1$, $\forall i \in \mathcal{I}$.

In addition, the transition probabilities in Markov chain are considered to be partially available, that is, some elements in matrix Λ are unknown. For instance, system (1) with four modes, the transition probability matrix Λ may be represented in the following form:

$$\Lambda = \begin{pmatrix} p_{11} & ? & ? & ? \\ p_{21} & ? & ? & ? \\ ? & p_{32} & ? & ? \\ ? & ? & p_{43} & ? \end{pmatrix} \tag{3}$$

where "?" stands for the unknown element. For notation clarity, we denote that for any $i \in \mathcal{I}$

$$\mathcal{I}_k^i \triangleq \{j : p_{ij} \text{ is known}\},\ \mathcal{I}_{uk}^i \triangleq \{j : p_{ij} \text{ is unknown}\}. \tag{4}$$

To ease the presentation, in the following, we denote $A(r(k)), r(k) = i$ by A_i. The same notation will also be used for $A_d(r(k))$, $B(r(k))$, $B_w(r(k))$, $C(r(k))$ and $E(r(k))$.

Consider the uncertainties in system (1), we assume that

$$A(r(k)) = \bar{A}(r(k)) + \Delta A(r(k))$$
$$A_d(r(k)) = \bar{A}_d(r(k)) + \Delta A_d(r(k))$$

where $\bar{A}(r(k))$ and $\bar{A}_d(r(k))$, for $r(k) = i, i \in \mathcal{I}$, are known real-valued constant matrices of appropriate dimensions that describe the nominal system. While $\Delta A(r(k))$ and $\Delta A_d(r(k))$, are unknown matrices representing the time-varying parameter uncertainties satisfying the following form:

$$\begin{aligned}(\Delta A(r(k)) \quad & \Delta A_d(r(k))) \\ &= G_1(r(k))\Delta r(k)(H_1(r(k)) \quad H_2(r(k)))\end{aligned}$$

where $G_1(r(k))$, $H_1(r(k))$ and $H_2(r(k))$ are known real constant matrices and $\Delta r(k)$ are unknown time-varying matrices satisfying the following conditions:

$$\|\Delta r(k)\| \leq I,\ \forall k \in \mathcal{Z} \text{ and } \forall r(k) = i,\ i \in \mathcal{I}$$

Consider the quantization effect, it is assumed that the measurement signals will be quantized before they are transmitted via the networks wherever data packet dropout or not.
The set of quantized levels is described as $\mathcal{U} = \{\pm u_i, u_i = \rho^i u_0, \pm 1, \pm 2, \ldots\} \cup \{0\}$, $0 < \rho < 1$, $u_0 > 0$, and the logarithmic quantizer $q(\cdot)$ as in Fu and Xie (2005) is applied

$$q(v) = \begin{cases} u_i, & \text{if } \frac{1}{1+\delta}\rho^i u_0 < v \leq \frac{1}{1-\delta}\rho^i u_0; \\ 0, & \text{if } v = 0; \\ -q(-v), & \text{if } v < 0. \end{cases} \tag{5}$$

where the parameter ρ is termed as quantization density, and $\delta = \frac{1-\rho}{1+\rho}$. From Fu and Xie (2005) we have

$$q(v) = (1 + \triangle_k)v \tag{6}$$

where $\triangle_k \in [-\delta, \delta]$, which is a suitable model for the logarithmic quantizer $q(v)$ with parameter δ.

Consider the quantizing effects are transformed into sector bounded uncertainties, associated to system (1), the quantized output with the packet dropout $y_c(k)$ is designed as

$$y_c(k) = \theta_k q(y(k)) = \theta_k(I + \Delta_k)y(k) \tag{7}$$

where the matrix $\Delta_k \triangleq diag\{\Delta_1, \Delta_2 \ldots\}$ satisfies $\|\Delta_k\| \leq \delta$. Let $\theta_k = 1$ indicate the the packet containing the measurement $y(k)$ has been successfully delivered to the state estimation center; while $\theta_k = 0$ corresponds to the dropout of the packet, and θ_k is independent Bernoulli distributed with probability distribution as follows:

$$Prob\{\theta_k = 1\} = \mathbb{E}\{\theta_k\} = \theta$$

In this paper, we assume that the accurate value of the system mode is available. The full-order Markov jump linear filter is given as follows:

$$\begin{aligned}\hat{x}(k+1) &= A_{fi}\hat{x}(k) + B_{fi}y_c(k) \\ \hat{z}(k) &= E_{fi}\hat{x}(k)\end{aligned} \tag{8}$$

where $\hat{x}(k) \in \mathbb{R}^n$ is the filter state; $\hat{z}(k) \in \mathbb{R}^q$ is the filter output; A_{fi}, B_{fi} and A_{fi} are filter matrices to be determined. Combining (1), (7) and (8), the filter error system is obtained as follows:

$$\begin{aligned}\xi(k+1) =& \tilde{A}_i\xi(k) + \tilde{A}_{di}\sum_{i=1}^{p}\lambda\{T_m = \tau_i\}x(k-\tau_i) \\ &+ \tilde{B}_{wi}w(k) \\ \tilde{z}(k) =& \tilde{E}_i\xi(k)\end{aligned} \tag{9}$$

where

$$\xi(k)=\begin{bmatrix} x(k)^T & \hat{x}(k)^T \end{bmatrix}^T, \quad \tilde{z}(k)=z(k)-\hat{z}(k)$$
$$\tilde{A}_i=\begin{bmatrix} A_i & 0 \\ \theta_k B_{fi}(I+\Delta_k)C_i & A_{fi} \end{bmatrix}, \quad \tilde{A}_{di}=\begin{bmatrix} A_{di} \\ 0 \end{bmatrix}$$
$$\tilde{B}_{wi}=\begin{bmatrix} B_{wi} \\ 0 \end{bmatrix}, \quad \tilde{E}_i=\begin{bmatrix} E_i & -E_{fi} \end{bmatrix}.$$

In order to proceed with the main results, we first introduce the following definitions and lemmas, which will be essential for the development of our main results.

Definition 1. Shi et al. (1999) System (9) is exponentially mean square stable if for every initial state $(x(0), r(0))$, there exist constants $0<\alpha<1$ and $\beta>0$ such that for all $k \geq 0$

$$\mathbb{E}\{\|x(k)\|^2 | x(0), r(0)\} \leq \beta\alpha^k\|x(0)\|^2 \tag{10}$$

Definition 2. Given the disturbance input $w(k) \in L_2$, a scalar $\gamma>0$, system (9) is exponentially mean square stable and with an H_∞ performance level γ if the following two conditions are satisfied:

1. When $w(k)=0$, system (9) is exponentially mean square stable in the sense of Definition 1.
2. When $w(k)\neq 0$, under zero initial conditions ,the following inequality holds

$$\mathbb{E}\{\sum_{k=0}^{\infty}\|\tilde{z}(k)\|^2\} < \gamma^2\|w(k)\|^2 \tag{11}$$

Hence, the main objective of this paper is to design the Markov jump filter for networked control system (1), such that the filter error system (9) is exponentially mean square stable with an H_∞ performance level γ in the presence of output quantization, distributed delay and packet dropout.

Lemma 1. *For any vectors $x, y \in \mathbb{R}^n$, matrices D, E and F with appropriate dimensions, and any scalar $\varepsilon>0$, if $F^TF \leq I$, then*

$$DFE + E^TF^TD^T \leq \varepsilon DD^T + \varepsilon^{-1}E^TE \tag{12}$$

3 Main Results

In this section, we first present, sufficient conditions to ensure system (9) is exponentially mean square stable.

Theorem 1. *Considering system (9), when w(k)=0, for a given quantization density $0<\rho<1$ and packet dropout rate $0<\theta<1$, it is exponentially mean square stable, if there exist matrices A_{fi}, B_{fi}, E_{fi}, $P_i>0$, $i\in\mathcal{I}$ and $Q>0$ satisfying:*

$$\Phi_i=\begin{bmatrix} \phi_{11i} & * \\ \phi_{21i} & \phi_{22i} \end{bmatrix}<0 \tag{13}$$

where

$$\phi_{11i}=diag\{-P_i+Q, -P_i, -Q\}$$
$$\phi_{21i}=\begin{bmatrix} A_i & 0 & A_{di} \\ \theta B_{fi}(I+\Delta_k)C_i & B_{fi} & 0 \end{bmatrix}$$
$$\phi_{22i}=diag\{-\bar{P}_i^{-1}, -\bar{P}_i^{-1}\}, \quad \bar{P}_i=\sum_{j=1}^{N}p_{ij}P_j.$$

Proof:

Construct the following Lyapunov functional candidate for system (9) as

$$V(x(k),r(k)) = V_1(x(k),r(k)) + V_2(x(k),r(k))$$
$$V_1(x(k),r(k)) = \xi^T(k)P(r(k))\xi(k)$$
$$V_2(x(k),r(k)) = \sum_{i=1}^{P}\lambda\{T_m=\tau_i\}\sum_{s=k-\tau_i}^{k-1}x^T(s)Qx(s) \tag{14}$$

Take the mathematical expectation $\mathbb{E}\{\Delta V(k)\} \triangleq \mathbb{E}\{V(x(k+1), r(k+1))|x(k), r(k)\} - V(x(k), r(k))$, then for each $r(k)=i, i\in\mathcal{I}$, we obtain

$$\mathbb{E}\{\Delta V_1(k)\} = (\tilde{A}_i\xi(k)+\tilde{A}_{di}\sum_{i=1}^{p}\beta_i x(k-\tau_i))^T\phi_{22i}^{-1} \times (\tilde{A}_i\xi(k)+\tilde{A}_{di}\sum_{i=1}^{p}\beta_i x(k-\tau_i)) - \xi^T(k)P_i\xi(k)$$

$$\begin{aligned}\mathbb{E}\{\Delta V_2(k)\} &= \sum_{i=1}^{p}\beta_i x^T(k)Qx(k) - \sum_{i=1}^{p}\beta_i x^T(k-\tau_i)Qx(k-\tau_i) \\ &\leq x^T(k)Qx(k) - \sum_{i=1}^{p}\beta_i x^T(k-\tau_i)Q\sum_{i=1}^{p}\beta_i x^T(k-\tau_i)\end{aligned} \tag{15}$$

A combination of (15) leads to

$$\mathbb{E}\{\Delta V(k)\} \leq \eta^T(k)\bar{\Phi}_i\eta(k)$$

where

$$\eta^T(k) = [\ \xi^T(k) \quad \sum_{i=1}^{p}\beta_i x^T(k-\tau_i)\ ]$$
$$\bar{\Phi}_i = \phi_{11i} + \phi_{21i}^T\phi_{22i}^{-1}\phi_{21i}$$

by Schur Lemma, we can get $\bar{\Phi}_i \le \Phi_i$, that means

$$\mathbb{E}\{\Delta V(k)\} \le 0$$

Let $\alpha \triangleq 1 - min_{r(k)\in\mathcal{I}}\{\frac{\lambda_{min}(\Phi(r(k)))}{\lambda_{max}(\Phi(r(k)))}\} < 1$, thus

$$\frac{\mathbb{E}\{V(x(k+1), r(k+1))|x(k), r(k)\} - V(x(k), r(k))}{V(x(k), r(k))}$$
$$\le \alpha - 1$$

Note that $\Phi_i < 0$, that is, $0 < \alpha < 1$, such that

$$\mathbb{E}\{V(x(k+1), r(k+1))|x(k), r(k)\} \le \alpha V(x(k), r(k))$$

from [13], let $\beta \triangleq max_{i,j\in\mathcal{I}}\{\frac{\lambda_{max}(\Phi_i)}{\lambda_{min}(\Phi_j)}\} < 1$, we have

$$\mathbb{E}\|x(k)\|^2 \le \alpha^k\beta\|x(0)\|^2$$

Therefore, by Definition 1, it can be verified that when $w(k) = 0$, system (9) is exponentially mean square stable. This completes the proof.

Remark 1. Although a sufficient condition is given by Theorem 1 to ensure system (9) is exponentially mean square stable, it is difficult to apply it into system (9) with partially unknown transition probabilities. In the following, a technique will be developed to cope with such problem for system (9) with partially unknown transition probabilities. Whatever for any $i \in \mathcal{I}, \mathcal{I}_{uk}^i = \emptyset$, the considered system is the one with completely known transition probabilities, or $i \in \mathcal{I}, \mathcal{I}_k^i = \emptyset$, all the transition probabilities are unaccessible in the considerer system, the condition is still valid.

Lemma 2. *Consider system (9) with partially unknown transition probabilities, when w(k)=0, for a given quantization density* $0 < \rho < 1$ *and packet dropout rate* $0 < \theta < 1$*, system (9) is exponentially mean square stable, if there exist matrices* A_{fi}, B_{fi}, E_{fi}, $P_i > 0$, $i \in \mathcal{I}$ *and* $Q > 0$ *satisfying:*

$$\tilde{\Phi}_i = \begin{bmatrix} \phi_{11i} & * \\ \tilde{\phi}_{21i} & \tilde{\phi}_{22i} \end{bmatrix} < 0 \qquad (16)$$

where

$$\tilde{\phi}_{22i} = diag\{-\tilde{P}_i^{-1}, -\tilde{P}_i^{-1}\}$$
$$\tilde{P}_i = \sum_{j\in I_k^i}^{N} p_{ij}P_j + (1 - \sum_{j\in\mathcal{I}_k^i}^{N} p_{ij})\sum_{j\in\mathcal{I}_{uk}^i}^{N} P_j.$$

Proof:

The desired result can be worked out by the following fact

$$\bar{P}_i = \sum_{j=1}^{N} p_{ij}P_j = \sum_{j\in\mathcal{I}_k^i}^{N} p_{ij}P_j + \sum_{j\in\mathcal{I}_{uk}^i}^{N} p_{ij}P_j \le \tilde{P}_i. \quad (17)$$

Next we will present a condition to ensure system (9) to be exponentially mean square stable with a given performance level.

Theorem 2. *Considering system (9), for a given quantization density* $0 < \rho < 1$*, packet dropout rate* $0 < \theta < 1$ *and* $\gamma > 0$*, it is exponentially mean square stable with an* H_∞ *performance level* γ *under zero initial condition, if there exist matrices* A_{fi}, B_{fi}, E_{fi}, $P_i > 0$, $i \in \mathcal{I}$ *and* $Q > 0$*, scalars* $\varepsilon_{1i} > 0$ *and* $\varepsilon_{2i} > 0$, $i \in \mathcal{I}$ *satisfying:*

$$\Omega_i = \begin{bmatrix} \psi_{11i} & * & * & * \\ \psi_{21i} & \psi_{22i} & * & * \\ \psi_{31i} & \psi_{32i} & \psi_{33i} & * \\ \psi_{41i} & \psi_{42i} & 0 & \psi_{44i} \end{bmatrix} < 0 \qquad (18)$$

where

$$\psi_{11i} = diag\{-P_i + Q, -P_i, -Q, -\gamma^2 I\}$$
$$\psi_{22i} = diag\{-\tilde{P}_i^{-1}, -\tilde{P}_i^{-1}, -I\}$$
$$\psi_{21i} = \begin{bmatrix} A_i & 0 & A_{di} & B_{wi} \\ \theta B_{fi}C_i & A_{fi} & 0 & 0 \\ E_i & E_{fi} & 0 & 0 \end{bmatrix}$$
$$\psi_{31i} = \begin{bmatrix} \varepsilon_{1i}H_{1i} & 0 & \varepsilon_{1i}H_{2i} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\psi_{32i} = \begin{bmatrix} 0 & 0 & 0 \\ G_{1i}^T & 0 & 0 \end{bmatrix}$$
$$\psi_{41i} = \begin{bmatrix} \varepsilon_{2i}\theta C_i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \psi_{42i} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & B_{fi}^T & 0 \end{bmatrix}$$
$$\psi_{33i} = diag\{-\varepsilon_{1i}, -\varepsilon_{1i}\}, \quad \psi_{44i} = diag\{-\varepsilon_{2i}, \frac{-\varepsilon_{2i}}{\delta^2}I\}$$

Proof:

First, by Theorem 1, system (9) with $w(k) = 0$ is exponentially mean square stable, So what we need to do is to show when $w(k) \ne 0$, system (9) has an H_∞ performance level γ.

According to Theorem 1, we can obtain

$$\mathbb{E}\{\tilde{z}^T(k)\tilde{z}(k)\} - \gamma^2 w^T(k)w(k)$$
$$< \mathbb{E}\{\tilde{z}^T(k)\tilde{z}(k)\} - \gamma^2 w^T(k)w(k) + \mathbb{E}(\Delta V(k))$$
$$< \mathbb{E}\{\tilde{E}_i^T\xi(k)\tilde{E}_i\xi(k)\} - \gamma^2 w^T(k)w(k) + \xi^T(k)\Phi_i\xi(k)$$

by Schur Lemma, Define

$$\delta^T(k) = \left[\begin{array}{ccc} \xi^T(k) & \sum_{i=1}^{p}\beta_i x^T(k-\tau_i) & w^T(k) \end{array}\right]$$

by (15), we can obtain

$$\mathbb{E}\{\tilde{z}^T(k)\tilde{z}(k)\} - \gamma^2 w^T(k)w(k) < \delta^T(k)\bar{\Omega}_i\delta(k)$$

where

$$\bar{\Omega}_i = \begin{bmatrix} \phi_{11i} & * & * & * \\ 0 & -\gamma^2 I & * & * \\ \phi_{21i} & \tilde{B}_{wi} & \phi_{22i} & * \\ \tilde{E}_i & 0 & 0 & -I \end{bmatrix} \quad (19)$$

According to (17), we obtain the following one holds,

$$\bar{\Omega}_i < \tilde{\Omega}_i = \begin{bmatrix} \phi_{11i} & * & * & * \\ 0 & -\gamma^2 I & * & * \\ \phi_{21i} & \tilde{B}_{wi} & -\tilde{\phi}_{22i} & * \\ \tilde{E}_i & 0 & 0 & -I \end{bmatrix} \quad (20)$$

Bearing in mind all admissible uncertainties of system (9), $\tilde{\Omega}_i$ can be written as

$$\tilde{\Omega}_i = \begin{bmatrix} \psi_{11i} & * \\ \psi_{21i} & \psi_{22i} \end{bmatrix} + \begin{bmatrix} 0 & * & * & * & * & * & * \\ 0 & 0 & * & * & * & * & * \\ 0 & 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & 0 & * & * & * \\ G_{1i}\Delta_i H_{1i} & 0 & G_{1i}\Delta_i H_{2i} & 0 & 0 & * & * \\ \theta B_{fi}\Delta_k C_i & 0 & 0 & 0 & 0 & 0 & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (21)$$

In view of Lemma 1, we obtain that (20) holds, if and only if there exist scalars $\varepsilon_{1i} > 0$ and $\varepsilon_{2i} > 0$, such that

$$\begin{aligned} \tilde{\Omega}_i < \hat{\Omega}_i = & \begin{bmatrix} \psi_{11i} & * \\ \psi_{21i} & \psi_{22i} \end{bmatrix} \\ & + \varepsilon_{1i}^{-1}\left[\begin{array}{ccccccc} 0 & 0 & 0 & 0 & G_{1i}^T & 0 & 0 \end{array}\right]^T \\ & \times \left[\begin{array}{ccccccc} 0 & 0 & 0 & 0 & G_{1i}^T & 0 & 0 \end{array}\right] \\ & + \varepsilon_{1i}\left[\begin{array}{ccccccc} \tilde{H}_{1i} & 0 & H_{2i} & 0 & 0 & 0 & 0 \end{array}\right]^T \\ & \times \left[\begin{array}{ccccccc} \tilde{H}_{1i} & 0 & H_{2i} & 0 & 0 & 0 & 0 \end{array}\right] \\ & + \varepsilon_{2i}^{-1}\left[\begin{array}{ccccccc} 0 & 0 & 0 & 0 & 0 & B_{fi}^T & 0 \end{array}\right]^T \\ & \times \Delta_k^2\left[\begin{array}{ccccccc} 0 & 0 & 0 & 0 & 0 & B_{fi}^T & 0 \end{array}\right] \\ & + \varepsilon_{2i}\left[\begin{array}{ccccccc} \theta C_i & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]^T \\ & \times \left[\begin{array}{ccccccc} \theta C_i & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{aligned} \quad (22)$$

due to

$$\Delta_k^2 \leq \delta^2$$

By Schur Lemma, if matrix inequality (22) holds, $\Omega_i < 0$ is equivalent to $\hat{\Omega}_i < 0$. Then

$$\mathbb{E}\{\tilde{z}^T(k)\tilde{z}(k)\} - \gamma^2 w^T(k)w(k) < \delta^T(k)\bar{\Omega}_i\delta(k) < 0 \quad (23)$$

Taking sum of both sides of (23) from $k = 0$ to ∞, and recalling that $x(0) = 0$, the following inequality holds

$$\mathbb{E}\{\sum_{k=0}^{\infty}\tilde{z}^T(k)\tilde{z}(k)\} < \gamma^2\sum_{k=0}^{\infty}w^T(k)w(k)$$

Therefore, by Definition 2, system (9) with distributed delay, quantization and packet dropout is robust exponentially mean square stable with an H_∞ performance level γ.

It is noted that the matrix (18) in Theorem 2 is not a linear one, thus it cannot be solved directly by Matlab LMI Toolbox. To this end, we will convert (18) into an LMI. Also the sufficient condition for the robust H_∞ filter gain matrices A_{fi} and B_{fi} will be designed in the following Theorem.

Theorem 3. *Considering system (9), for a given quantization density $0 < \rho < 1$, packet dropout rate $0 < \theta < 1$ and $\gamma > 0$, it is exponentially mean square stable with an H_∞ performance level γ under zero initial condition, if there exist matrices Y_{1i}, Y_{2i}, E_{fi}, $P_i > 0$, $i \in \mathcal{I}$ and $Q > 0$, scalars $\varepsilon_{1i} > 0$ and $\varepsilon_{2i} > 0$, $i \in \mathcal{I}$ satisfying:*

$$\Omega_i = \begin{bmatrix} \psi_{11i} & * & * & * \\ \bar{\psi}_{21i} & \bar{\psi}_{22i} & * & * \\ \psi_{31i} & \bar{\psi}_{32i} & \psi_{33i} & * \\ \psi_{41i} & \bar{\psi}_{42i} & 0 & \psi_{44i} \end{bmatrix} < 0 \quad (24)$$

where

$$\bar{\psi}_{22i} = diag\{-\tilde{P}_i, -\tilde{P}_i, -I\}$$

$$\bar{\psi}_{21i} = \begin{bmatrix} \tilde{P}_i A_i & 0 & \tilde{P}_i A_{di} & \tilde{P}_i B_{wi} \\ \theta Y_{2i} C_i & Y_{1i} & 0 & 0 \\ E_i & E_{fi} & 0 & 0 \end{bmatrix}$$

$$\bar{\psi}_{32i} = \begin{bmatrix} 0 & 0 & 0 \\ G_{1i}^T\tilde{P}_i^T & 0 & 0 \end{bmatrix}, \quad \bar{\psi}_{42i} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & Y_{2i}^T & 0 \end{bmatrix}$$

Furthermore, if the above conditions have feasible solutions, the H_∞ filter parameters A_{fi}, B_{fi}, E_{fi} can be computed via the following:

$$A_{fi} = \tilde{P}_i^{-1}Y_{1i}, \quad B_{fi} = \tilde{P}_i^{-1}Y_{2i}, \quad E_{fi} = E_{ifeas}$$

Proof:

We define the following matrices

$$Y_{1i} = \tilde{P}_i A_{fi} \text{ and } Y_{2i} = \tilde{P}_i B_{fi} \tag{25}$$

where Y_{1i} and Y_{2i} are non-singular matrices with appropriate dimensions. Multiplying $diag\{I, I, I, I, \tilde{P}_i, \tilde{P}_i, I, I, I, I, I\}$ and its transpose on the left-hand and the right-hand side of (18), respectively, and rewrite the parameters in Theorem 2. Hence, from (25), the filter parameters can be solved as $A_{fi} = \tilde{P}_i^{-1} Y_{1i}$ and $B_{fi} = \tilde{P}_i^{-1} Y_{2i}$. This completes the proof.

Remark 2. Both Theorems 2 and 3 provide conditions to design a filter which can guarantee the resulting filtering error system to be exponentially mean square stable with a prescribed H_∞ performance level. Due to the uncertainties and unknown transition probabilities, only sufficient conditions are derived, which has certain conservativeness, and should be further looked at in our future work.

4 Examples

In this section, a numerical example is given to show the usefulness of the results obtained in the previous section.

Consider system (9) with the following parameters. There are four modes of distributed time delays in the system, their probabilities are $Prob\{\tau_1 = 1\} = \beta_1 = 0.3$, $Prob\{\tau_2 = 2\} = \beta_2 = 0.4$, $Prob\{\tau_3 = 3\} = \beta_3 = 0.2$, and $Prob\{\tau_4 = 4\} = \beta_4 = 0.1$. Assume that the quantization density $\rho = 0.4$, and the disturbance is a Gaussian white noise. The initial condition is selected as $x(0) = [0,\ 0]^T$ and $\hat{x}(0) = [0.1,\ 0.1]^T$.

In addition, the system has three modes that means $\mathcal{I} = \{1, 2, 3\}$, and the mode switching governed by partially unknown transition probabilities is supposed to be

$$\begin{pmatrix} 0.1 & ? & ? \\ 0.4 & ? & ? \\ 0.5 & ? & ? \end{pmatrix}$$

and the other parameters are set as follows:

$$\bar{A}_1 = \begin{bmatrix} -2 & 0 \\ 0.6 & -1 \end{bmatrix} \quad \bar{A}_2 = \begin{bmatrix} -1 & 1 \\ -0.6 & -3.3 \end{bmatrix}$$

$$\bar{A}_3 = \begin{bmatrix} -1.5 & 0.2 \\ 0.5 & -1.1 \end{bmatrix} \quad \bar{A}_{d1} = \begin{bmatrix} -1 & 0.3 \\ -1 & -1 \end{bmatrix}$$

$$\bar{A}_{d2} = \begin{bmatrix} -1 & -1.3 \\ 0.7 & -2.1 \end{bmatrix} \quad \bar{A}_{d3} = \begin{bmatrix} -0.8 & 0.6 \\ 0.8 & 0.8 \end{bmatrix}$$

$$B_{w1} = \begin{bmatrix} 0.1 & 0.1 \\ 0 & 0.1 \end{bmatrix} \quad B_{w2} = \begin{bmatrix} 0.1 & 0.1 \\ 0 & 0.1 \end{bmatrix}$$

$$B_{w3} = \begin{bmatrix} 0.1 & 0 \\ 0.1 & -0.1 \end{bmatrix} \quad C_1 = \begin{bmatrix} 3 & -7 \\ 2.4 & 3.9 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 3.1 & 4.1 \\ -1.1 & 5.4 \end{bmatrix} \quad C_3 = \begin{bmatrix} 2.4 & -2.1 \\ 1.1 & 3.1 \end{bmatrix}$$

$$E_1 = \begin{bmatrix} -0.1 & -0.2 \end{bmatrix} \quad E_2 = \begin{bmatrix} 0.1 & -1.1 \end{bmatrix}$$

$$E_3 = \begin{bmatrix} -0.7 & -0.3 \end{bmatrix} \quad H_{11} = \begin{bmatrix} 0.1 & 0 \\ 0.1 & 0.1 \end{bmatrix}$$

$$H_{12} = \begin{bmatrix} 0.1 & 0 \\ 0.1 & -0.1 \end{bmatrix} \quad H_{13} = \begin{bmatrix} 0.1 & 0 \\ 0.1 & -0.1 \end{bmatrix}$$

$$H_{21} = \begin{bmatrix} 0.4 & 0.1 \\ -0.1 & 0.1 \end{bmatrix} \quad H_{22} = \begin{bmatrix} 0.4 & 0.1 \\ 0 & -0.1 \end{bmatrix}$$

$$H_{23} = \begin{bmatrix} 0.4 & 0.1 \\ -0.1 & -0.1 \end{bmatrix} \quad G_{11} = \begin{bmatrix} 0 & 0 \\ 0 & 0.1 \end{bmatrix}$$

$$G_{12} = \begin{bmatrix} 0 & 0 \\ 0.1 & 0 \end{bmatrix} \quad G_{13} = \begin{bmatrix} 0 & 0 \\ 0 & 0.1 \end{bmatrix}$$

By applying Theorem 3, the optimal value for H_∞ performance $\gamma = 1.2492$, and the filter matrices can be computed as:

$$A_{f1} = \begin{bmatrix} 2.5998 & -0.4516 \\ -0.8804 & 0.9342 \end{bmatrix}$$

$$A_{f2} = \begin{bmatrix} 3.4982 & -4.6226 \\ -2.5060 & 16.3507 \end{bmatrix}$$

$$A_{f3} = \begin{bmatrix} 8.7629 & 1.4122 \\ -0.0883 & 0.9118 \end{bmatrix}$$

$$B_{f1} = \begin{bmatrix} 1.6173 & -0.6481 \\ -0.6706 & 1.9519 \end{bmatrix}$$

$$B_{f2} = \begin{bmatrix} 2.9926 & -0.3111 \\ -0.9038 & 0.4789 \end{bmatrix}$$

$$B_{f3} = \begin{bmatrix} 2.5831 & -1.9722 \\ -2.2534 & 2.6240 \end{bmatrix}$$

$$E_{f1} = \begin{bmatrix} 0.0011 & 0.0017 \end{bmatrix}$$

$$E_{f2} = \begin{bmatrix} -0.0005 & 0.0043 \end{bmatrix}$$

$$E_{f3} = \begin{bmatrix} -0.0068 & 0.0015 \end{bmatrix}$$

The simulation result of Markov chain is shown in Fig.1. There are three modes in the results, which are stochastic with partially unknown probabilities. The system state $x(k)$ and the filtering state $\hat{x}$ are plotted in Fig.2. It should be pointed out that the results reported in Yang and Han (2013), time-delay, quantization and packet drop out are not all considered in systems. In our current work, we have shown that the system with these practical multiple network-induced phenomenon is still mean square stable under some reasonable conditions. In addition, we also consider the system modeling error in design, so our results have better robustness at the same time. Fig.3 displays the filtering error $\tilde{z}(k)$ converges to zero. The output $y(k)$ and quantized output $q(y(k))$ are shown in Fig.4. From these figures, it can be seen that the designed filter meets the specified requirements. At the same time it can also be seen that our approach is able to deal with multiple network-induced phenomenons.

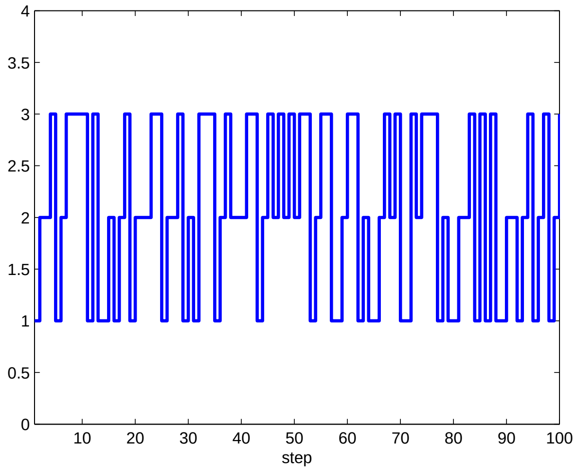

Figure 1: Parameters change of r(k)

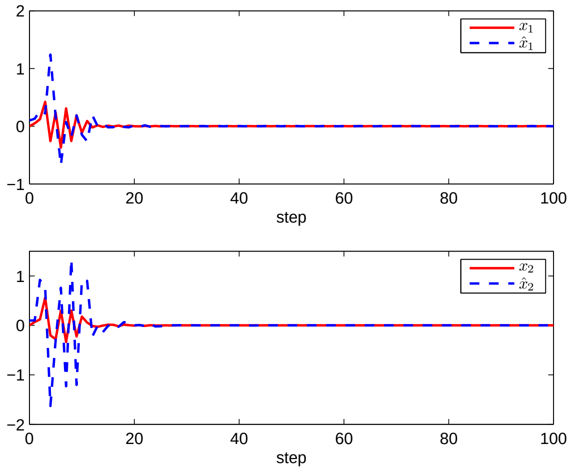

Figure 2: The system state $x(k)$ and the filtering state $\hat{x}$.

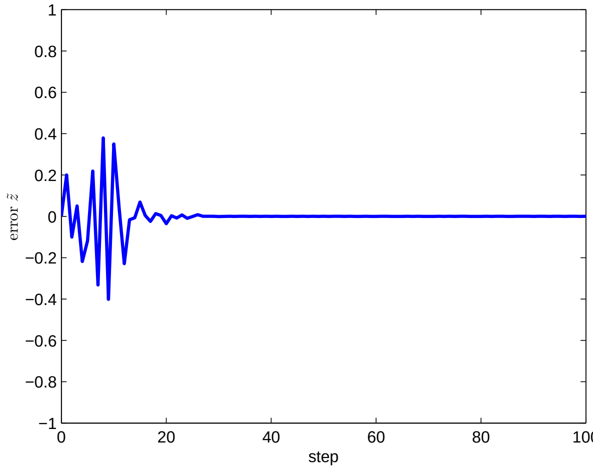

Figure 3: The filtering error $\tilde{z}(k)$.

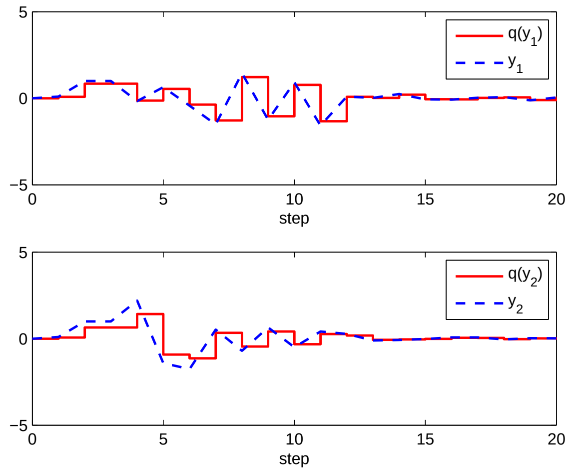

Figure 4: The output $y(k)$ and the quantized output $q(y(k))$.

5 Conclusion

In this paper, we have presented a new approach on H_∞ filtering problem for uncertain network control system with distributed delay, quantization and packet dropout. The occurrence probability of each time delay is considered in the system, and the output signals are quantized before they are communicated. The packet dropout have Bernoulli distributions. Markovian jump linear systems with the partially unknown transition probabilities are adopted to model the system. Based on the new model, sufficient conditions are developed for the robust mean square stability of the filtering error system with a given H_∞ performance. Then the robust Markov jump H_∞ filter is derived in terms of strict LMIs. A numerical example shows the effectiveness of the obtained approach. Further research work will be focused on developing methods to NCSs with nonlinear plant, possibly by fuzzy modeling approach.

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