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H_∞ robust controller design for the synchronization of master-slave chaotic systems with disturbance input

Bo Wang [1] Peng Shi [2] Hamid Reza Karimi [3] Jun Wang [1]

[1] School of Electrical and Information Engineering, Xihua University, Chengdu, 610096, China.
School of Applied Mathematics, University Electronic Science and Technology of China, Chengdu 610054, China.
E-mail: bowang@uestc.edu.cn

[2] Department of Computing and Mathematical Sciences,University of Glamorgan, Pontypridd, CF37 1DL, United Kingdom. Corresponding Author E-mail: pshi@glam.ac.uk
School of Engineering and Science, Victoria University, Melbourne, Vic 8001, Australia.

[3] Department of Engineering, Faculty of Engineering and Science, University of Agder, N-4898 Grimstad, Norway.
E-mail: hamid.r.karimi@uia.no

Abstract

This paper is concerned with the robust control problems for the synchronization of master-slave chaotic systems with disturbance input. By constructing a series of Lyapunov functions, novel H_∞ robust synchronization controllers are designed, whose control regulation possess the characteristic of simpleness and explicitness. Finally, numerical simulations are provided to demonstrate the effectiveness of the proposed techniques.

Keywords: Chaos; Synchronization; Robust control; Disturbance

1 Introduction

Chaos synchronization is first introduced in Fujisaka and Yamada (1983), and get much more attention from researchers Pecora and Carroll (1990). The idea of synchronization is to design a controller so that the output of the slave system can follow the output of the master system asymptotically with time passing by. In general, there are four classes of synchronization for dynamical systems: (i) identical or complete synchronization, (ii) generalized synchronization, (iii) phase synchronization, (iv) anticipated and lag synchronization and amplitude envelope synchronization, see Luo (2009). Due to its powerful applications in chemical reactions, power converters, biological systems, information processing, secure communications, chaos synchronization has been developed extensively. During the last decade, many techniques for handling chaos synchronization have been studied. In Huang et al. (2009), based on the sliding mode control technique, a single controller is designed to achieve chaos synchronization of four-dimensional energy resource systems. In Sun (2009), via the time-domain approach, a tracking control is proposed to realize chaos synchronization for the uncertain Genesio-Tesi chaotic systems with deadzone nonlinearity. In Wu et al. (2009), through

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adopting active control method, backstepping design and adaptive method, the synchronization problems for a new $3D$ chaotic system are discussed. In Chen (2009), a linear balanced feedback gain control method is then employed to design a controller to achieve the global synchronization of two identical four-scroll Liu chaotic systems. In Kuntanapreeda (2009), based on Lyapunov stability theory and linear matrix inequality formulation, a simple linear feedback control law is obtained to make the state of two identical unified chaotic systems asymptotically synchronized. In Effa et al. (2009), the problems on chaos synchronization between chaotic Colpitts oscillators are investigated. Recently, the problem of synchronization for different class of master-slave systems with time-delays and uncertainties are studied in (Karimi and Gao (2010), Karimi et al. (2012), Karimi (2012), Karimi (2011)) and the references therein. However, those synchronization methods usually are specialized for one typical chaotic system, which limit their application in practice. In this paper, we consider the following chaotic system:

$$\begin{array}{l} \dot{z}_i(t) = z_{i+1}(t), i = 1, 2, \cdots, n-1 \\ \dot{z}_n(t) = f(z(t)) \end{array} \tag{1}$$

where $Z(t) = (z_1(t), z_2(t), \cdots, z_n(t))^T$ is the system state vector.

Such a model can represent many chaotic systems. Actually, through topological transformation, many existing chaotic systems, such as Chen systems, Lorenz systems, Lü systems, etc, can be transformed as the form of system (1). Compared with the researches focus on one chaotic system, the investigation towards chaotic system (1) will have wider range of practical application. In addition, disturbance is common in real control system. Usually H_∞ method is utilized to deal with such problem and corresponding investigation can be seen in (Basin et al. (2011), Song et al. (2009), Yang et al. (2011a), Zhang et al. (2008), He et al. (2009), Zhang and Shi (2009), Ahn and Song (2011), Shi et al. (2012), Liu et al. (2011), Yang et al. (2011b)) and the references therein. Therefore, in this paper, based on Lyapunov function and linear matrix inequality, H_∞ controller design for synchronization of the chaotic system (1) with disturbance input will be studied. Corresponding simulation results will be given to illustrate the usefulness of theoretical results obtained.

Notation: The notations used throughout the paper are fairly standard. Let R^n be the n-dimensional Euclidean space, $R^{n\times m}$ the set of $n \times m$ real matrix, $*$ the symmetric part in a matrix, I the identity matrix with appropriate dimensions, diag$\{\cdots\}$ the diagonal matrix. By $A > 0$ we mean that A is a real symmetric positive definitive matrix.

2 System description and preliminaries

In real world, the order of chaotic system (1) usually will not go beyond fourth order. Therefore, we first consider the following fourth order master-slave chaotic system.

$$\begin{array}{l} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = x_3(t) \\ \dot{x}_3(t) = x_4(t) \\ \dot{x}_4(t) = f(x(t)) + w(t) + u(t) \end{array} \tag{2}$$

where $x(t) = (x_1(t), x_2(t), x_3(t), x_4(t))^T$ is the state vector of the slave system,$f(x(t))$ is the nonlinear function variable of the slave system, $w(t) \in L_2[t_0, \infty]$ is the disturbance input, $u(t)$ is the control input, has an effect on slave system through $\dot{x}(t)$.

$$\begin{array}{l} \dot{y}_1(t) = y_2(t) \\ \dot{y}_2(t) = y_3(t) \\ \dot{y}_3(t) = y_4(t) \\ \dot{y}_4(t) = g(y(t)) \end{array} \tag{3}$$

where $y(t) = (y_1(t), y_2(t), y_3(t), y_4(t))^T$ is the state vector of the master system, $g(y(t))$ is nonlinear function variable of the master system.

Define the tracking error vector as follow

$$E(t) = x(t) - y(t)$$

The error dynamical system model can be described by

$$\begin{aligned} &\dot{e}_1(t) = e_2(t) \\ &\dot{e}_2(t) = e_3(t) \\ &\dot{e}_3(t) = e_4(t) \\ &\dot{e}_4(t) = F(x(t), y(t)) + w(t) + u(t) \\ &F(x(t), y(t)) = f(x(t)) - g(y(t)) \\ &h(t) = C^T E(t) + dw(t) \end{aligned} \tag{4}$$

where $C = (c_1, c_2, c_3, c_4)^T$ is a vector, d is a positive scalar, $E(t) = (e_1(t), e_2(t), e_3(t), e_4(t))^T$ is the tracking error vector, and $h(t)$ is the error output.

In the paper, the following definition is needed:

Definition 1: Under the assumption of zero initial condition, the systems (2) can be synchronized to system (3) with H_∞ norm bound γ, if there exists any nonzero $w(t) \in L_2[t_0, \infty]$ such that

$$\|h(t)\|_2 \leq \gamma \|w(t)\|_2$$

3 Main Results

In this section, based on Lyapunov method and linear matrix inequality, the following theorem can be concluded.

Theorem 1. System (2) with any initial conditions can be synchronized to system (3) by the following H_∞ controller

$$\begin{aligned} &u(t) = \\ &-(((k_1k_2 + 1)k_3 + k_1)k_4 + k_1k_2 + 1)e_1(t) \\ &-((k_1k_2 + (k_1 + k_2)k_3 + 2)k_4 \\ &+(k_1k_2 + 1)k_3 + 2k_1 + k_2)e_2(t) \\ &-((k_1 + k_2 + k_3)k_4 \\ &+(k_1k_2 + (k_1 + k_2)k_3 + 3))e_3(t) \\ &-(k_1 + k_2 + k_3 + k_4)e_4(t) - F(x(t), y(t)) \end{aligned} \tag{5}$$

$$\begin{bmatrix} CC^T - K & dC + B \\ * & d^2 - \gamma^2 \end{bmatrix} < 0 \tag{6}$$

where $B = (0, 0, 0, 1)^T$, $K = diag\{k_1, k_2, k_3, k_4\}$ and the control gains k_1, k_2, k_3, k_4 are positive scalars to be determined.

Proof. Choose the first Lyapunov functional candidate as

$$V_1(t) = \frac{1}{2}z_1^2(t)$$

where

$$z_1(t) = e_1(t)$$

The time derivative of $V_1(t)$ along trajectories of error model (4) is

$$\begin{aligned} &\dot{V}_1(t) = z_1(t)e_2(t) \\ &= -k_1z_1^2(t) + z_1(t)(k_1e_1(t) + e_2(t)) \end{aligned}$$

Choose the second Lyapunov functional candidate as

$$V_2 = \frac{1}{2}z_1^2(t) + \frac{1}{2}z_2^2(t)$$

where

$$z_2(t) = k_1e_1(t) + e_2(t)$$

The time derivative of $V_2(t)$ along trajectories of error model (4) is

$$\begin{aligned} &\dot{V}_2(t) = \dot{V}_1(t) + z_2(t)(k_1e_2(t) + e_3(t)) \\ &= -k_1z_1^2(t) - k_2z_2^2(t) + z_2(t)((k_1k_2 \\ &+1)e_1(t) + (k_1 + k_2)e_2(t) + e_3(t)) \end{aligned}$$

Choose the third Lyapunov functional candidate as

$$V(t) = \frac{1}{2}z_1^2(t) + \frac{1}{2}z_2^2(t) + \frac{1}{2}z_3^2(t)$$

where

$$z_3(t) = (k_1k_2 + 1)e_1(t) + (k_1 + k_2)e_2(t) + e_3(t)$$

The time derivative of $V(t)$ along trajectories of error model (4) is

$$\begin{aligned} &\dot{V}_3(t) = \dot{V}_2(t) + z_3(t)((k_1k_2 + 1)e_2(t) \\ &+(k_1 + k_2)e_3(t) + F(x(t), y(t)) + u(t) \\ &+w(t)) \\ &= -k_1z_1^2(t) - k_2z_2^2(t) - k_3z_3^2(t) \\ &+z_3(t)(((k_1k_2 + 1)k_3 + k_1)e_1(t) \\ &+(k_1k_2 + (k_1 + k_2)k_3 + 2)e_2(t) \\ &+(k_1 + k_2 + k_3)e_3(t) + F(x(t), y(t)) \\ &+u(t) + w(t)) \end{aligned}$$

Choose the fourth Lyapunov functional candidate as

$$V(t) = \frac{1}{2}z_1^2(t) + \frac{1}{2}z_2^2(t) + \frac{1}{2}z_3^2(t) + \frac{1}{2}z_4^2(t)$$

where

$$\begin{aligned} &z_4(t) = ((k_1k_2 + 1)k_3 + k_1)e_1(t) \\ &+(k_1k_2 + (k_1 + k_2)k_3 + 2)e_2(t) \\ &+(k_1 + k_2 + k_3)e_3(t) + e_4(t) \end{aligned}$$

The time derivative of $V(t)$ along trajectories of error model (4) is

$$
\begin{aligned}
&\dot{V}(t) = \dot{V}_3(t) \\
&+z_4(t)(((k_1k_2+1)k_3+k_1)e_2(t) \\
&+(k_1k_2+(k_1+k_2)k_3+2)e_3(t) \\
&+(k_1+k_2+k_3)e_4(t)+F(x(t),y(t))+u(t)) \\
&= -k_1z_1^2(t)-k_2z_2^2(t)-k_3z_3^2(t)-k_4z_4^2(t) \\
&+z_4(((k_1k_2+1)k_3+k_1)k_4+k_1k_2+1)e_1(t) \\
&+((k_1k_2+(k_1+k_2)k_3+2)k_4 \\
&+(k_1k_2+1)k_3+2k_1+k_2)e_2(t) \\
&+((k_1+k_2+k_3)k_4 \\
&+(k_1k_2+(k_1+k_2)k_3+3))e_3(t) \\
&+(k_1+k_2+k_3+k_4)e_4(t)+F(x(t),y(t)) \\
&+u(t)+w(t))
\end{aligned} \tag{7}
$$

Substituting the control law (5) into (7) results in

$$\dot{V} = -Z^T(t)KZ(t)+z_4(t)w(t)$$

Consider the following performance index

$$
\begin{aligned}
&J = \int_{t_0}^{t_T}[h^T(t)h(t)-\gamma^2w^T(t)w(t)]dt \\
&= \int_{t_0}^{t_T}[h^T(t)h(t)-\gamma^2w^T(t)w(t)+\dot{V}(t)]dt \\
&+V(t_0)-V(t_T)
\end{aligned}
$$

For $V(t_0)=0$ and $V(t_T)\geq 0$, we have

$$
\begin{aligned}
&J \leq \int_{t_0}^{t_T}[h^T(t)h(t)-\gamma^2w^T(t)w(t)+\dot{V}(t)]dt \\
&= \int_{t_0}^{t_T}\eta^T(t)\Omega\eta(t)dt
\end{aligned}
$$

where

$$\eta(t) = [E^T(t), w^T(t)]^T$$

$$\Omega = \begin{bmatrix} CC^T-K & dC+B \\ * & d^2-\gamma^2 \end{bmatrix}$$

Therefore, if LMI (6) is satisfied, then $J\leq 0$ for any nonzero $w(t)\in L_2[t_0,\infty]$. This completes the proof.

4 Further results

In this section, we first consider the following second order master-slave chaotic systems

$$
\begin{aligned}
&\dot{x}_1(t) = x_2(t) \\
&\dot{x}_2(t) = f(x(t))+w(t)+u(t)
\end{aligned} \tag{8}
$$

where $x(t)=(x_1(t),x_2(t))^T$ and

$$
\begin{aligned}
&\dot{y}_1(t) = y_2(t) \\
&\dot{y}_2(t) = g(y(t))
\end{aligned} \tag{9}
$$

with $y(t)=(y_1(t),y_2(t))^T$.

Based on Theorem 1, the following corollary can be deduced.

Corollary 1. System (8) with any initial conditions can be synchronized to system (9) by the following H_∞ controller

$$
\begin{aligned}
&u(t) = -(k_1k_2+1)e_1(t)-(k_1+k_2)e_2(t) \\
&-F(x(t),y(t))
\end{aligned} \tag{10}
$$

$$\begin{bmatrix} CC^T-K & dC+B \\ * & d^2-\gamma^2 \end{bmatrix} < 0 \tag{11}$$

where $B=(0,1)^T$, $K=diag\{k_1,k_2\}$ and the control gains k_1,k_2 are positive scalars to be determined.

Next, we consider the following third order master-slave chaotic systems

$$
\begin{aligned}
&\dot{x}_1(t) = x_2(t) \\
&\dot{x}_2(t) = x_3(t) \\
&\dot{x}_3(t) = f(x(t))+w(t)+u(t)
\end{aligned} \tag{12}
$$

where $x(t)=(x_1(t),x_2(t),x_3(t))^T$ and

$$
\begin{aligned}
&\dot{y}_1(t) = y_2(t) \\
&\dot{y}_2(t) = y_3(t) \\
&\dot{y}_3(t) = g(y(t))
\end{aligned} \tag{13}
$$

with $y(t)=(y_1(t),y_2(t),y_3(t))^T$.

Based on Theorem 1, the following corollary can be concluded.

Corollary 2. System (12) with any initial conditions can be synchronized to system (13) by the following H_∞ controller

$$
\begin{aligned}
&u(t) = -((k_1k_2+1)k_3+k_1)e_1(t) \\
&-(k_1k_2+(k_1+k_2)k_3+2)e_2(t) \\
&-(k_1+k_2+k_3)e_3(t)-F(x(t),y(t))
\end{aligned} \tag{14}
$$

$$\begin{bmatrix} CC^T-K & dC+B \\ * & d^2-\gamma^2 \end{bmatrix} < 0 \tag{15}$$

where $B=(0,0,1)^T$, $K=diag\{k_1,k_2,k_3\}$ and the control gains k_1,k_2,k_3 are positive scalars to be determined.

5 Simulation results

For numerical simulation, we first consider the following third order master-slave chaotic systems

$$\begin{aligned}
&\dot{y}_1(t) = y_2(t)\\
&\dot{y}_2(t) = y_3(t)\\
&\dot{y}_3(t) = g(y(t))\\
&g(y(t)) = 5.5y_1(t) - 3.5y_2(t) - y_3(t) + y_1^3(t)
\end{aligned}$$

and

$$\begin{aligned}
&\dot{x}_1(t) = x_2(t)\\
&\dot{x}_2(t) = x_3(t)\\
&\dot{x}_3(t) = f(x(t)) + w(t) + u(t)\\
&f(x(t)) = -1.2x_1(t) - 1x_2(t) - 0.6x_3(t) + x_1^2(t)
\end{aligned}$$

where

$$\begin{aligned}
&\gamma = 0.4, d = 0.1, C = [1.1, 0.6, 0.7]\\
&x_0 = [-1, 0, -3]^T,\\
&y_0 = [0.1922, -3.4031, 4.3610]^T
\end{aligned}$$

$$w(t) = \begin{cases} 15\sin(2t)\cos(e^t), & t \geq 10s \\ 0, & else \end{cases}$$

According to the given control regulation, we get $K = diag\{5.3383, 2.9184, 15.7688\}$. The numerical simulation results can be seen in Figures 1-3.

Next, we consider the following second order master-slave chaotic systems

$$\begin{aligned}
&\dot{x}_1(t) = x_2(t)\\
&\dot{x}_2(t) = f(x(t)) + w(t) + u(t)\\
&f(x(t)) = -0.5x_2(t) + x_1(t) - 0.8x_1^3(t) - 2\cos(1.5t)
\end{aligned}$$

and

$$\begin{aligned}
&\dot{y}_1(t) = y_2(t)\\
&\dot{y}_2(t) = g(y(t))\\
&g(y(t)) = -0.4y_2(t) + 1.1y_1(t) - y_1^3(t) - 2.1\cos(1.8t)
\end{aligned}$$

where

$$\begin{aligned}
&\gamma = 0.3, d = 0.1, C = [0.8, 0.5]\\
&x_0 = [2, 3]^T, y_0 = [-2, 7]^T
\end{aligned}$$

$$w(t) = \begin{cases} 15\cos(2t)\sin(\frac{e^t}{t+1}), & t \geq 10s \\ 0, & else \end{cases}$$

According to the given control regulation, we get $K = diag\{2.0589, 15.6052\}$. The numerical simulation results can be seen in Figures 4-6.

Remark. Figures 1 and 4 depict the time response of system disturbance input, Figures 2 and 5 depict the

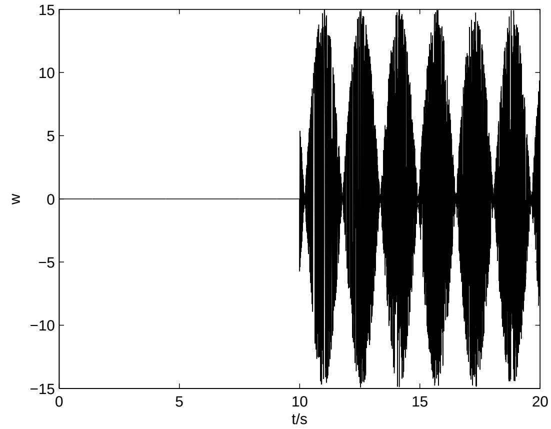

Figure 1: Time response of disturbance input of third order chaotic system

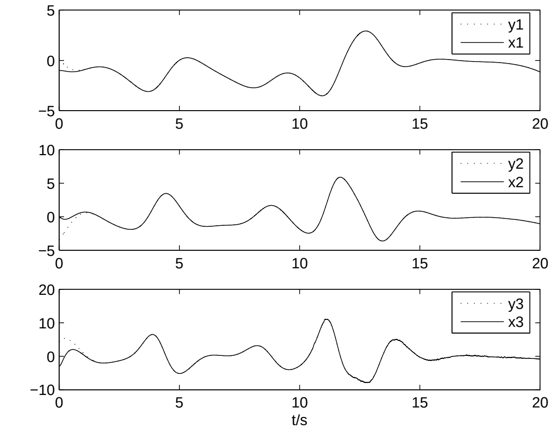

Figure 2: Time response of state variable of third order master-slave systems

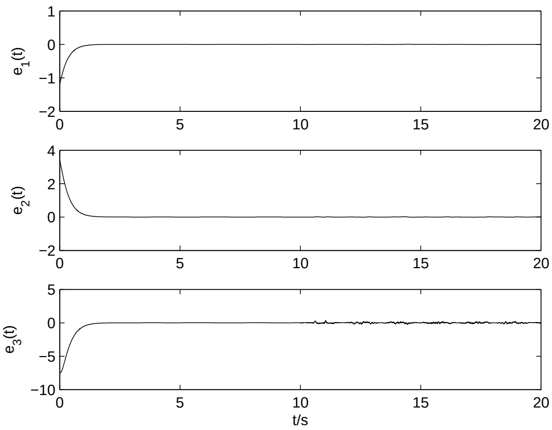

Figure 3: Time response of error variable of third order master-slave systems

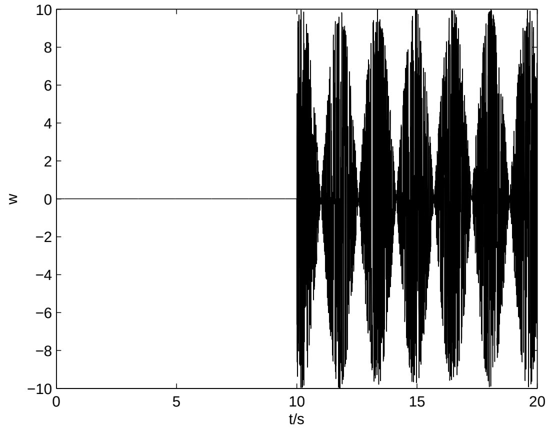

Figure 4: Time response of distance input of second order chaotic system

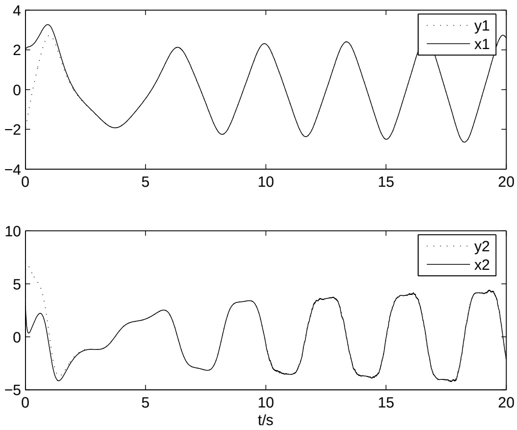

Figure 5: Time response of state variable of second order master-slave systems

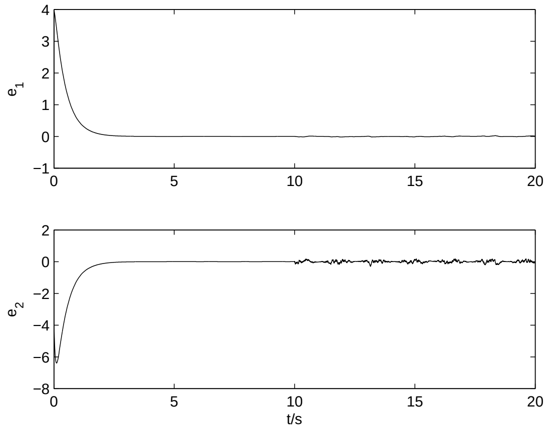

Figure 6: Time response of error variable of second order master-slave systems

time response of state variable of master-slave systems. The time responses of error variable of master-slave systems are plotted in Figures 3 and 6. It can see that both of the second order and the third order chaotic systems display complex dynamics. Based on H_∞ controller designed, we can see that in the early period, the slave system spends less than 1 second realizing the tracking with the master system; later the disturbance input is added at the 10th second, we can see the error variable of master-slave systems jitter in a small range, which satisfies the H_∞ performance index given. The simulation results demonstrate the effectiveness of the proposed techniques.

6 Conclusion

In this paper, the problems on robust control for the synchronization of master - slave chaotic systems with disturbance input have been studied. Based on Lyapunov method and LMI techniques, novel H_∞ robust synchronization controllers have been presented, whose control regulation possess the characteristic of simpleness and explicitness. Finally, some numerical simulations have been carried out to demonstrate the effectiveness of results obtained.

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