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Optimizing Adaptive Control Allocation With Actuator Dynamics

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Abstract

In this work we address the optimizing control allocation problem for an over-actuated nonlinear time-varying system with actuator dynamic where parameters affine in the actuator and effector model may be assumed unknown. Instead of optimizing the control allocation at each time instant, a dynamic approach is considered by constructing actuator reference update-laws that represent an asymptotically optimal allocation search. By using Lyapunov analysis for cascaded set-stable systems, uniform global/local asymptotic stability is guaranteed for the optimal equilibrium sets described by the system, the control allocation update-law and the adaptive update-law, if some persistence of exitation condition holds. Simulations of a scaled-model ship, manoeuvred at low-speed, demonstrate the performance of the proposed allocation scheme.*

Keywords: Control allocation; Adaptive control; Nonlinear systems

1 Introduction

Consider the high-level system dynamics

$$\dot{x} = f(t,x) + g(t,x)\tau \tag{1}$$

the effector model

$$\tau = \Phi(t,x,u,\theta) \tag{2}$$

$$\Phi(t,x,u,\theta) := \Phi_0(t,x,u) + \Phi_{\theta_2}(t,x,u)\theta_2 + \Phi_{\theta_1}(t,x,u)\theta_1 \tag{3}$$

and the actuator dynamics

$$\dot{u} = f_{u0}(t,x,u,u_{cmd}) + f_{u\theta}(t,x,u,u_{cmd})\theta_1 \tag{4}$$

where $t \geq 0$, $x \in \mathbb{R}^n$, $u \in \mathbb{R}^r$, $\tau \in \mathbb{R}^d$, $\theta := (\theta_1^{\mathrm{T}}, \theta_2^{\mathrm{T}})^{\mathrm{T}}$, $\theta_1 \in \mathbb{R}^{m_1}$, $\theta_2 \in \mathbb{R}^{m_2}$, $u_{cmd} \in \mathbb{R}^c$. The constant parameter vectors θ_2 and θ_1 contains parameters of the actuator and effector model, that will be viewed as uncertain parameters to be adapted. It is assumed that x and u are measured while τ is unknown, and u_{cmd} is the input.

This work is motivated by the over-actuated control allocation problem $d \leq r$, where the problem is described by a nonlinear system, divided into a dynamic high-level part (1), a dynamic low-level part (4) and a static part (2). Consider the static optimal control allocation problem:

$$\min_{u_d} J(t,x,u_d) \quad s.t. \quad \tau_c - \Phi(t,x,u_d+\tilde{u},\hat{\theta}) = 0, \tag{5}$$

where $\hat{\theta} := \left(\hat{\theta}_1^{\mathrm{T}}, \hat{\theta}_2^{\mathrm{T}}\right)^{\mathrm{T}}$ is the parameter estimates, $\tilde{u} := u - u_d$ and u_d is the actuator reference. The main contribution in this paper is an adaptive allocation algorithm that generates a desired reference u_d for the low-level control based on a high level control law τ_c, where (5) not necessarily needs to be solved exactly at each time instant.

Optimizing control allocation solutions have been derived for certain classes of over-actuated systems,

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such as aircraft, automotive vehicles and marine vessels, (Enns, 1998; Buffington et al., 1998; Sørdalen, 1997; Bodson, 2002; Härkegård, 2002; Luo et al., 2004, 2005; Poonamallee et al., 2005; Johansen et al., 2004) and (Johansen et al., 2005). The control allocation problem is, in (Enns, 1998; Buffington et al., 1998; Sørdalen, 1997; Johansen et al., 2005; Bodson, 2002) and (Härkegård, 2002), viewed as a *static* or *quasi-dynamic* problem considering non-adaptive linear effector models of the form $\tau = Gu$, neglecting the effect of actuator dynamics. In (Luo et al., 2004) and (Luo et al., 2005) a dynamic model predictive approach is considered to solve the allocation problem with linear time-varying dynamics in the actuator model, $T\dot{u}+u = u_{cmd}$. In (Poonamallee et al., 2005) and (Johansen et al., 2004) sequential quadratic programming techniques are used to cope with nonlinearities in the control allocation problem due to singularity avoidance. The main advantage of the control allocation approach is in general the modularity and the ability to handle redundancy and constraints. In the present work we consider dynamic solutions based on the ideas presented in (Johansen, 2004) and (Tjønnås and Johansen, 2005). In (Johansen, 2004) it was shown that it is not necessary to solve the optimization problem (5) exactly at each time instant. Further a control Lyapunov function was used to derive an exponentially convergent update-law for u (related to a gradient or Newton-like optimization) such that the control allocation problem (5) could be solved dynamically. It was also shown that convergence and asymptotic optimality of the system, composed by the dynamic control allocation and a uniform globally exponentially stable trajectory-tracking controller τ_c, guarantees uniform boundedness and uniform global exponential convergence to the optimal solution of the system. The advantage of this approach is computational efficiency and simplicity of implementation, since the optimizing control allocation algorithm is implemented as a dynamic nonlinear controller. Solving (5) online at each sampling instant requires a computationally more expensive numerical solution of a nonlinear program in order to guarantee optimality. In (Tjønnås and Johansen, 2005) the results were extended by allowing uncertain parameters, associated with an adaptive law, in the effector model, and by applying set-stability analysis in order to also conclude asymptotic stability of the optimal solution. The results in (Tjønnås and Johansen, 2005) are extended in (Tjønnås and Johansen, 2007) by considering actuator dynamic and relaxing some conditions using the theory in (Tjønnås et al., 2006). In the present paper we extend the result in (Tjønnås and Johansen, 2007) by a slightly different parameterization of (2) and (3).

Whenever referring to the notion of set-stability, the set has the property of being nonempty, and we strictly follow the definitions given in (Tjønnås et al., 2006) motivated by (Teel et al., 2002) and (Lin et al., 1996).

2 Adaptive control allocation with actuator dynamics

The task of the dynamic control allocation algorithm is to connect the high and low level controls by taking the desired virtual control τ_c as an input and computing the desired actuator reference u_d as an output. Based on the minimization problem (5) where J is a cost function that incorporates objectives such as minimum power consumption and actuator constraints (implemented as barrier functions), the Lagrangian function

$$L(t,x,u_d,\tilde{u},\lambda,\hat{\theta}) := J(t,x,u_d) + (\tau_c - \Phi(t,x,u_d+\tilde{u},\hat{\theta}))^{\mathrm{T}}\lambda \quad (6)$$

can be introduced. The idea is then to define update laws for the actuator reference u_d and the Lagrangian parameter λ, based on a Lyapunov approach, such that u_d and λ converges to a set defined by the first order optimal condition for L.

Since the parameter vector θ from the effector and actuator models are unknown, an adaptive update law for $\hat{\theta}$ is defined. The parameter estimates are used in the Lagrangian function (6) and a certainty equivalent adaptive optimal control allocation can be defined. The following observers are used in order to produce estimates of the parameters:

$$\dot{\hat{u}} = A_{\hat{u}}(u-\hat{u}) + f_{u0}(t,x,u,u_{cmd}) + f_{u\theta}(t,x,u,u_{cmd})\theta_1$$
$$\dot{\hat{x}} = A_{\hat{x}}(x-\hat{x}) + f(t,x) + g(t,x)\Phi(t,x,u,\hat{\theta}).$$

where $(-A_{\hat{u}})$ and $(-A_{\hat{x}})$ are Hurwitz matrices.

In the following, if stating that a function F is uniformly bounded by y, this means that there exist a function $G_F : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ such that $|F(t,y,z)| < G_f(|y|)$ for all y, z and t.

Assumpiton 1 *(Plant)*

a) *The states from (1) and (4) are known for all t.*

b) *The function f is uniformly locally Lipschitz in x and uniformly bounded by x. The function g is uniformly bounded and it's partial derivatives are bounded by x.*

c) *The function Φ is twice differentiable and uniformly bounded by x and u. Moreover it's partial derivatives are uniformly bounded by x.*

d) *There exists constants $\varrho_2 > \varrho_1 > 0$, such that $\forall t$, x, u and θ*

$$\varrho_1 I \leq \frac{\partial \Phi}{\partial u}(t,x,u,\theta)\left(\frac{\partial \Phi}{\partial u}(t,x,u,\theta)\right)^T \leq \varrho_2 I. \quad (7)$$

Assumpiton 2 *(High and Low level Controller Algorithms)*

a) *There exists a high level control $\tau_c := k(t,x)$, that render the equilibrium of (1) UGAS for $\tau = \tau_c$. The function k is uniformly bounded by x and differentiable. It's partial derivatives are uniformly bounded by x.*

b) *There exists a low-level control $u_{cmd} := k_u(t,x,u,u_d,\dot{u}_d,\hat{\theta}_1)$ that makes the equilibrium of*

$$\dot{\tilde{u}} = f_{\tilde{u}}(t,x,\tilde{u},u_d,\hat{\theta}_1,\theta_1) \quad (8)$$

UGAS if $\hat{\theta}_1 = \theta_1$ and x, u_d, $\dot{u}_d$ exist for all $t > 0$, where

$$\begin{aligned} & f_{\tilde{u}}(t,x,\tilde{u},u_d,\hat{\theta}_1,\theta_1) := \\ & + f_{u0}(t,x,u,k_u(t,x,u,u_d,\dot{u}_d(t),\hat{\theta}_1)) \\ & + f_{u\theta}(t,x,u,k_u(t,x,u,u_d,\dot{u}_d(t),\hat{\theta}_1))\theta_1 \\ & - k_u(t,x,u,u_d,\dot{u}_d(t),\hat{\theta}_1). \end{aligned}$$

Remark 1 *From assumption 2a) there exist a Lyapunov function $V_x : \mathbb{R}_{\geq 0} \times \mathbb{R}^n \mapsto \mathbb{R}_{\geq 0}$ and $\mathcal{K}_\infty$ functions α_{x1}, α_{x2}, α_{x3} and α_{x4} such that*

$$\alpha_{x1}(|x|) \leq V_x(t,x) \leq \alpha_{x2}(|x|) \quad (9)$$

$$\frac{\partial V_x}{\partial t} + \frac{\partial V_x}{\partial x}\left(f(t,x) + g(t,x)k(t,x)\right) \leq -\alpha_{x3}(|x|) \quad (10)$$

$$\left|\frac{\partial V_x}{\partial x}\right| \leq \alpha_{x4}(|x|). \quad (11)$$

We will not discuss the details in these assumptions, but they are sufficient in order to guarantee existence of solutions and validity of the update-laws that we propose in this paper, see (Tjønnås and Johansen, 2005). The main problem formulation is given by:

Problem: Define update-laws (14)-(16) for u_d, λ and $\hat{\theta}$, such that the stability of the closed loop:

$$\begin{aligned} \dot{x} &= f(t,x) + g(t,x)k(t,x) \\ &\quad + g(t,x)\left(\Phi(t,x,u,\theta) - k(t,x)\right) && (12) \\ \dot{\tilde{u}} &= f_{\tilde{u}}(t,x,\tilde{u},u_d,\hat{\theta}_1,\theta_1) && (13) \\ \dot{u}_d &:= f_d(t,x,\tilde{u},u_d,\hat{\theta}) && (14) \\ \dot{\lambda} &:= f_\lambda(t,x,\tilde{u},u_d,\hat{\theta}) && (15) \\ \dot{\hat{\theta}} &:= -f_{\hat{\theta}}(t,x,\tilde{u},u_d,\hat{\theta}) && (16) \\ \dot{\eta}_u &= -A_{\hat{u}}\eta_u + \bar{f}_{u\theta}(t,x,u_d,\tilde{u},\hat{\theta})\tilde{\theta}_1 && (17) \\ \dot{\eta}_x &= -A_{\hat{x}}\eta_x + \Phi_{\theta_2}(t,x,u)\tilde{\theta}_2 + \Phi_{\theta_1}(t,x,u)\tilde{\theta}_1 && (18) \end{aligned}$$

where $\bar{f}_{u\theta}(t,x,u_d,\tilde{u},\hat{\theta}) := f_{u\theta}(t,x,u,k_u(t,x,u,u_d, f_d(t,x,\tilde{u},u_d,\hat{\theta}),\hat{\theta}_1))$, $\tilde{\theta} = \theta - \hat{\theta}$, $\eta_u := u - \hat{u}$, $\eta_x := x - \hat{x}$, is conserved and $u_d(t)$ converges to an optimal solution with respect to the minimization problem (5).

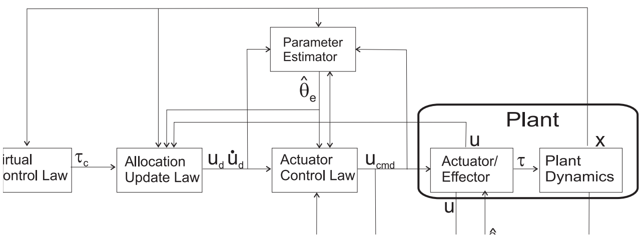

Figure 1: The closed loop diagram of the certainty equivalent control allocation algorithm

Let (12) define the sub-system Σ_1 and (13)-(18) define the sub-system Σ_2, then Σ_1 and Σ_2 form a cascade as long as $x(t)$ exists for all $t > 0$, and is viewed as a time-varying input to Σ_2. For the system Σ_2 we will consider stability with respect to the set

$$\mathcal{O}_{u_d\lambda\tilde{\theta}}(t,x) := \left\{ z_{u_d\lambda\tilde{\theta}} \in \mathbb{R}^{n_{u_d\lambda\tilde{\theta}}} \,\middle|\, f_{\mathcal{O}_{u\lambda\tilde{\theta}}}(t,x,z_{u_d\lambda\tilde{\theta}}) = 0 \right\} \quad (19)$$

where $n_{u_d\lambda\tilde{\theta}} := 3r + d + n + m$, $z_{u_d\lambda\tilde{\theta}} := \left(u_d^{\mathrm{T}}, \lambda^{\mathrm{T}}, \tilde{u}^{\mathrm{T}}, \eta_u^{\mathrm{T}}, \eta_x^{\mathrm{T}}, \tilde{\theta}^{\mathrm{T}}\right)^{\mathrm{T}}$ and $f_{\mathcal{O}_{u\lambda\tilde{\theta}}}(t,x,z_{u_d\lambda\tilde{\theta}}) := \left(\left(\frac{\partial L}{\partial u}\right)^{\mathrm{T}}, \left(\frac{\partial L}{\partial \lambda}\right)^{\mathrm{T}}, \tilde{u}^{\mathrm{T}}, \eta_u^{\mathrm{T}}, \eta_x^{\mathrm{T}}, \tilde{\theta}^{\mathrm{T}}\right)$. In order to relate the notion of optimal control allocation to the set $\mathcal{O}_{u_d\lambda\tilde{\theta}}(t,x)$, we introduce the sufficient conditions for the set

$$\mathcal{O}_{u_d\lambda}(t,x,\tilde{u},\hat{\theta}) := \left\{ \left(u_d^{\mathrm{T}}, \lambda^{\mathrm{T}}\right)^{\mathrm{T}} \in \mathbb{R}^{r+d} \,\middle|\, \left(\left(\frac{\partial L}{\partial u_d}\right)^{\mathrm{T}}, \left(\frac{\partial L}{\partial \lambda}\right)^{\mathrm{T}}\right)^{\mathrm{T}} = 0 \right\}$$

to be the optimal solution of problem (5), by the following assumption.

Assumpiton 3 *(Optimal Control Allocation)*

a) *The cost function $J : \mathbb{R}_{\geq t_0} \times \mathbb{R}^{n \times r} \to \mathbb{R}$ is twice differentiable and $J(t,x,u_d) \to \infty$ as $|u_d| \to \infty$. Furthermore $\frac{\partial J}{\partial u_d}$, $\frac{\partial^2 J}{\partial t \partial u_d}$ and $\frac{\partial^2 J}{\partial x \partial u_d}$ are uniformly bounded by x and u_d.*

b) *There exists constants $k_2 > k_1 > 0$, such that $\forall$ t, x, $\hat{\theta}$, $\tilde{u}$ and $\left(u_d^T, \lambda^T\right)^T \notin \mathcal{O}_{u_d\lambda}(t,x,\tilde{u},\hat{\theta})$*

$$k_1 I \leq \frac{\partial^2 L}{\partial u_d^2}(t,x,u_d,\tilde{u},\lambda,\hat{\theta}) \leq k_2 I. \quad (20)$$

If $\left(u_d^T, \lambda^T\right)^T \in \mathcal{O}_{u_d\lambda}(t, x, \tilde{u}, \hat{\theta})$ *the lower bound is replaced by* $\frac{\partial^2 L}{\partial u_d^2} \geq 0$

Lemma 1 *By Assumption 1 there exists continuous functions* ς_x, ς_{xu}, $\varsigma_u : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$, *such that*

$$|\Phi_{\theta 1}(t, x, \tilde{u} + u_d)| + |\Phi_{\theta 2}(t, x, \tilde{u} + u_d)|$$
$$\leq \varsigma_x(|x|)\varsigma_{xx}(|x|)\varsigma_{xu}(|\tilde{u}|) + \varsigma_x(|x|)\varsigma_u\left(\left|z_{u_d\lambda\tilde{\theta}}\right|_{\mathcal{O}_{u_d\lambda\tilde{u}\tilde{\theta}}}\right).$$

Assumpiton 2 *(continued)*

c) *There exists a* $\mathcal{K}_\infty$ *function* $\alpha_k : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$, *such that*

$$\alpha_k^{-1}(|x|)\alpha_{x3}(|x|) \geq \alpha_{x4}(|x|)\bar{\varsigma}_x(|x|)\ , \tag{21}$$

where $\bar{\varsigma}_x(|x|) := \max(1, \varsigma_x(|x|), \varsigma_x(|x|)\varsigma_{xx}(|x|))$.

We approach the problem formulation by i) defining a Lyapunov like function, $V_{u_d\lambda\tilde{u}\eta\tilde{\theta}}$, for the system Σ_2 and defining explicit update-laws for u_d, λ and $\tilde{\theta}$ such that $\dot{V}_{u_d\lambda\tilde{u}\eta\tilde{\theta}} \leq 0$. ii) Furthermore, boundedness of the closed-loop system, Σ_1 and Σ_2 can be proved, and the cascade lemma from (Tjønnås et al., 2006) can be applied to prove convergence and stability.

Consider the Lyapunov function candidate

$$V_{u_d\lambda\tilde{u}\eta\tilde{\theta}}(t,x,u_d,\lambda,\tilde{u},\eta) := V_{\tilde{u}}(t,\tilde{u}) + \frac{1}{2}\eta_u^{\mathrm{T}}\Gamma_\eta\eta_u + \frac{1}{2}\eta_x^{\mathrm{T}}\Gamma_{\tilde{x}}\eta_x + \frac{1}{2}\left(\frac{\partial L^{\mathrm{T}}}{\partial u_d}\frac{\partial L}{\partial u_d} + \frac{\partial L^{\mathrm{T}}}{\partial \lambda}\frac{\partial L}{\partial \lambda}\right) + \frac{1}{2}\tilde{\theta}_1^{\mathrm{T}}\Gamma_{\theta_1}\tilde{\theta}_1 + \frac{1}{2}\tilde{\theta}_2^{\mathrm{T}}\Gamma_{\theta_2}\tilde{\theta}_2 \tag{22}$$

and the algorithm:

$$\begin{pmatrix} \dot{u}_d \\ \dot{\lambda} \end{pmatrix} = -\Gamma\mathbb{H}\begin{pmatrix} \frac{\partial L_{\hat{\theta}}}{\partial u_d} \\ \frac{\partial L_{\hat{\theta}}}{\partial \lambda} \end{pmatrix} - u_{ff} \tag{23}$$

$$\begin{aligned}\dot{\hat{\theta}}_1^{\mathrm{T}} &= \left(\frac{\partial V_{\tilde{u}}}{\partial \tilde{u}} + \eta_u^{\mathrm{T}}\Gamma_\eta\right) f_{u\theta}(t, x, u_d + \tilde{u}, u_{cmd})\Gamma_{\theta_1}^{-1} \\ &+ \left(\tilde{x}^{\mathrm{T}}\Gamma_{\tilde{x}} + \frac{\partial L^{\mathrm{T}}}{\partial u_d}\frac{\partial^2 L}{\partial \tilde{u}\partial u_d} + \frac{\partial L^{\mathrm{T}}}{\partial \lambda}\frac{\partial^2 L}{\partial \tilde{u}\partial \lambda}\right) f_{u\theta}(t, x, u, u_{cmd})\Gamma_{\theta_1}^{-1} \\ &+ \left(\frac{\partial L^{\mathrm{T}}}{\partial u_d}\frac{\partial^2 L}{\partial x\partial u_d} + \frac{\partial L^{\mathrm{T}}}{\partial \lambda}\frac{\partial^2 L}{\partial x\partial \lambda}\right) g(t,x)\Phi_{\theta_1}(t, x, u_d + \tilde{u})\Gamma_{\theta_1}^{-1}\end{aligned} \tag{24}$$

$$\begin{aligned}\dot{\hat{\theta}}_2^{\mathrm{T}} &= \eta_x^{\mathrm{T}}\Gamma_{\tilde{x}} g(t,x)\Phi_{\theta_2}(t,x,u)\Gamma_{\theta_2}^{-1} \\ &+ \left(\frac{\partial L^{\mathrm{T}}}{\partial u_d}\frac{\partial^2 L}{\partial x\partial u_d} + \frac{\partial L^{\mathrm{T}}}{\partial \lambda}\frac{\partial^2 L}{\partial x\partial \lambda}\right) g(t,x)\Phi_{\theta_2}(t,x,u)\Gamma_{\theta_2}^{-1}\end{aligned} \tag{25}$$

where $\mathbb{H} := \begin{pmatrix} \frac{\partial^2 L}{\partial u_d^2} & \frac{\partial^2 L}{\partial \lambda \partial u_d} \\ \frac{\partial^2 L}{\partial u_d \partial \lambda} & 0 \end{pmatrix}$, Γ is a possibly time-varying symmetric positive definite weighting matrix and u_{ff} is a feed-forward like term:

$$\begin{aligned} u_{ff} &:= \mathbb{H}^{-1}\begin{pmatrix} \frac{\partial^2 L}{\partial t\partial u_d} \\ \frac{\partial^2 L}{\partial t\partial \lambda} \end{pmatrix} + \mathbb{H}^{-1}\begin{pmatrix} \frac{\partial^2 L}{\partial x\partial u_d} \\ \frac{\partial^2 L}{\partial x\partial \lambda} \end{pmatrix} f(t,x) \\ &+ \mathbb{H}^{-1}\begin{pmatrix} \frac{\partial^2 L}{\partial x\partial u_d} \\ \frac{\partial^2 L}{\partial x\partial \lambda} \end{pmatrix} g(t,x)(k(t,x) - \Phi(t,x,u_d+\tilde{u},\hat{\theta})) \\ &+ \mathbb{H}^{-1}\begin{pmatrix} \frac{\partial^2 L}{\partial \tilde{u}\partial u_d} \\ \frac{\partial^2 L}{\partial \tilde{u}\partial \lambda} \end{pmatrix} f_{\tilde{u}}(t,x,\tilde{u},u_d,u_{cmd},\hat{\theta}) + \mathbb{H}^{-1}\begin{pmatrix} \frac{\partial^2 L}{\partial \hat{\theta}\partial u_d} \\ \frac{\partial^2 L}{\partial \hat{\theta}\partial \lambda} \end{pmatrix}\dot{\hat{\theta}}, \end{aligned}$$

if $\det(\mathbb{H}) \neq 0$ and $u_{ff} := 0$ if $\det(\mathbb{H}) = 0$, then the time derivative of $V_{u_d\lambda\tilde{u}\eta\tilde{\theta}}$ along the trajectories of Σ_1 and Σ_2 is given by:

$$\begin{aligned}\dot{V}_{u_d\lambda\tilde{u}\eta\tilde{\theta}} &= -\eta^{\mathrm{T}}\Gamma_\eta A\eta - \alpha_{\tilde{u}3}(|\tilde{u}|) - \tilde{x}^{\mathrm{T}}\Gamma_{\tilde{x}}A_{\tilde{x}}\tilde{x} \\ &- \left(\frac{\partial L}{\partial u_d}^{\mathrm{T}}, \frac{\partial L}{\partial \lambda}^{\mathrm{T}}\right)\mathbb{H}\Gamma\mathbb{H}\left(\frac{\partial L}{\partial u_d}^{\mathrm{T}}, \frac{\partial L}{\partial \lambda}^{\mathrm{T}}\right)^{\mathrm{T}}.\end{aligned} \tag{26}$$

Proposition 1 *If the assumptions 1, 2 and 3 are satisfied, then the solution of the closed-loop (12)-(18) is bounded with respect to a set* $\mathcal{O}_{xu_d\lambda\tilde{\theta}}(t) := \mathcal{O}_{u_d\lambda\tilde{\theta}}(t,0) \times \{x \in \mathbb{R}_{\geq t_0} | x = 0\}$. *Furthermore the set* $\mathcal{O}_{xu_d\lambda\tilde{\theta}}$ *is UGS with respect to the system defined by (12)-(18). If in addition* $f_p(t) := f_{u\theta}(t, x(t), u(t), u_{cmd}(t))$ *and* $\Phi_g(t) := g(t, x(t))\Phi_{\theta_2}(t, x(t), u(t))$ *are Persistently Exited (PE), i.e. there exist constants* T *and* $\gamma > 0$, *such that*

$$\int_t^{t+T} F(\tau)^T F(\tau) d\tau \geq \gamma I\ , \quad \forall t > t_0,$$

is satisfied for $F(\tau) = f_p(t)$ *and* $F(\tau) = \Phi_g(t)$, *then the set* $\mathcal{O}_{xu_d\lambda\tilde{\theta}}$ *is UGAS with respect to the system (12)-(18).*

The proof of Proposition 1 involves similar steps as in the proof of the main result in (Tjønnås and Johansen, 2007) and is therefore omitted here.

Proposition 1 implies that the time-varying first order optimal set $\mathcal{O}_{xu\lambda\tilde{\theta}}(t)$ is uniformly stable, and in addition uniformly attractive if a PE assumption is satisfied. Thus adaptive optimal control allocation is achieved asymptotically for the closed loop under the PE condition.

Corollary 1 *If for* $\mathbb{U} \subset \mathbb{R}^r$ *there exist constant* $c_x > 0$ *such that for* $|x| \leq c_x$ *the domain* $\mathbb{U}_z \subset \mathbb{R}^n \times \mathbb{U} \times \mathbb{R}^{2r+d+n+m}$ *contain* $\mathcal{O}_{xu_d\lambda\tilde{\theta}}$, *then if the Assumptions 1-3 are satisfied, the set* $\mathcal{O}_{xu_d\lambda\tilde{\theta}}$ *is US with respect to the system (12)-(18). If in addition* $f_p(t)$ *and* $\Phi_g(t)$ *are PE,* $\mathcal{O}_{xu\lambda\tilde{\theta}}$ *is UAS with respect to the system (12)-(18).*

3 Example

In this section, simulation results of an over-actuated scaled-model ship, manoeuvred at low-speed, is presented. The scale model-ship is moved while experiencing disturbances caused by wind and current, and propellers trust losses. The propeller losses can be due to: *Axial Water Inflow, Cross Coupling Drag, Thruster-Hull* and *Thruster-Thruster Interaction* (see (Sørensen et al., 1997) and (Fossen and Blanke, 2000) for details). But in this example we limit our study to thruster loss caused by *Thruster-Hull interaction.* A 3DOF horizontal plane model described by:

$$\begin{aligned} \dot{\eta}_e &= R(\psi_p)\nu \\ \dot{\nu} &= -M^{-1}D\nu + M^{-1}\tau \\ \tau &= \Phi(\nu, u, \theta), \end{aligned} \tag{27}$$

is considered, where $\eta_e := (x_e, y_e, \psi_e)^{\mathrm{T}} := (x_p - x_d, y_p - y_d, \psi_p - \psi_d)^{\mathrm{T}}$ is the north and east positions and compass heading deviations. Subscript p and d denotes the actual and desired states. $\nu := (\upsilon_x, \upsilon_y, r)^{\mathrm{T}}$ is the body-fixed velocities, surge, sway and yaw, τ is the generalized force vector and $R(\psi_p)$ is the rotation matrix function between the body fixed and the earth fixed coordinate frame. The example we present here is based on (Lindegaard and Fossen, 2003), and is also studied in (Johansen, 2004), (Tjønnås and Johansen, 2005) and (Tjønnås and Johansen, 2007). In the considered model there are five force producing devices; the two main propellers aft of the hull, in conjunction with two rudders, and one tunnel thruster going through the hull of the vessel. ω_i denotes the propeller angular velocity and δ_i denotes the rudder deflection. $i = 1, 2$ denotes the aft actuators, while $i = 3$ denotes the tunnel thruster. Equation (27) can be rewritten in the form of (1) and (2) by:

$$x := (\eta_e, \nu)^{\mathrm{T}}, \ \theta_1 := (\theta_{11}, \theta_{12}, \theta_{13})^{\mathrm{T}}, \ \theta_2 := (\theta_{21}, \theta_{22}, \theta_{23})^{\mathrm{T}}$$

$$\tau := (\tau_1, \tau_2, \tau_3)^{\mathrm{T}}, \ u := (\omega_1, \omega_2, \omega_3, \delta_1, \delta_2)^{\mathrm{T}},$$

$$f := \begin{pmatrix} R(\psi_e + \psi_d)\nu \\ -M^{-1}D\nu \end{pmatrix}, \ g := \begin{pmatrix} 0 \\ M^{-1} \end{pmatrix},$$

$$\Phi(\nu, u, \theta) := G_u(u) \begin{pmatrix} T_1(\upsilon_x, \omega_1, \theta_{11}) \\ T_2(\upsilon_x, \omega_2, \theta_{12}) \\ T_3(\upsilon_x, \upsilon_y, \omega_3, \theta_{13}) \end{pmatrix} + R(\psi_p)\theta_2$$

$$G_u(u) := \begin{pmatrix} (1 - D_1) & (1 - D_2) & 0 \\ L_1 & L_2 & 1 \\ \Phi_{31} & \Phi_{32} & l_{3,x} \end{pmatrix}$$

$$\begin{aligned} \Phi_{31}(u) &:= -l_{1,y}(1 - D_1(u) + l_{1,x}L_1(u)), \\ \Phi_{32}(u) &:= -l_{2,y}\left(1 - D_2(u) + l_{2,x}L_2(u)\right). \end{aligned}$$

The thruster forces are given by:

$$T_i(\upsilon_x, \omega_i, \theta_{1i}) := T_{ni}(\omega_i) - \phi_i(\omega_i, \upsilon_x)\theta_{1i} \tag{28}$$

$$T_{ni}(\omega_i) := \begin{cases} k_{Tp_i}\omega_i^2 & \omega_i \geq 0 \\ k_{Tn_i}|\omega_i|\omega_i & \omega_i < 0 \end{cases},$$

$$\phi_1(\omega_1, \upsilon_x) := \omega_1\upsilon_x, \quad \phi_2(\omega_2, \upsilon_x) := \omega_2\upsilon_x$$

$$\phi_3(\omega_3) := \sqrt{\upsilon_x^2 + \upsilon_y^2)}\,|\omega_3|\,\omega_3, \quad \theta_{13} := k_{T\theta_3}$$

$$\theta_{11} := \begin{cases} k_{T\theta_1}(1 - w) & \upsilon_x \geq 0 \\ k_{T\theta_1} & \upsilon_x < 0 \end{cases},$$

$$\theta_{12} := \begin{cases} k_{T\theta_2}(1 - w) & \upsilon_x \geq 0 \\ k_{T\theta_2} & \upsilon_x < 0 \end{cases},$$

where $0 < w < 1$ is the wake fraction number, $\phi_i(\omega_i, \upsilon_x)\theta_{1i}$ is the thrust loss due to changes in the advance speed, $\upsilon_a = (1 - w)\upsilon_x$, and the unknown parameters θ_{1i} represents the thruster loss factors dependent on whether the hull invokes on the inflow of the propeller or not. The rudder lift and drag forces are projected through:

$$L_i(u) := \begin{cases} (1 + k_{Ln_i}\omega_i)(k_{L\delta1_i} + k_{L\delta2_i}|\delta_i|)\delta_i & ,\omega_i \geq 0 \\ 0 & ,\omega_i < 0 \end{cases},$$

$$D_i(u) := \begin{cases} (1 + k_{Dn_i}\omega_i)(k_{D\delta1_i}|\delta_i| + k_{D\delta2_i}\delta_i^2) & ,\omega_i \geq 0 \\ 0 & ,\omega_i < 0 \end{cases}.$$

Further more it is clear from (28) that $\Phi(\nu, u, \theta) = G_u(u)Q(u) + G_u(u)\phi(\omega, \upsilon_x)\theta_1 + R(\psi_e)\theta_2$, where $\phi(\omega, \upsilon_x) := diag(\phi_1, \phi_2, \phi_3)$, $Q(u)$ represents the nominal propeller thrust and θ_2 represents unknown external disturbances, such as ocean current, that are constant in the earth fixed coordinate frame.

The actuator error dynamic for each propeller is based on the propeller model presented in (Pivano et al., 2007) and given by

$$\begin{aligned} J_{mi}\dot{\tilde{\omega}}_i = &-k_{fi}\left(\tilde{\omega}_i + \omega_{di}\right) - \frac{T_{ni}}{a_T}(\tilde{\omega}_i + \omega_{di}) \\ &+ \frac{\phi_i(\omega_i, \upsilon_x)\theta_{1i}}{a_T} + u_{cmdi} - J_{mi}\dot{\omega}_{di} \end{aligned} \tag{29}$$

where $\tilde{\omega}_i := (\omega_i - \omega_{id})$, J_m is the shaft moment of inertia, k_f is a positive coefficient related to the viscous friction, a_T is a positive model constant (Pivano et al., 2006) and u_{cmd} is the commanded motor torque. By the quadratic Lyapunov function $\frac{\tilde{\omega}_i^2}{2}$ it is easy to see that the control law

$$\begin{aligned} u_{cmdi} := &- K_{\omega p}(\tilde{\omega}_i) - \frac{\phi_i(\omega_i, \upsilon_x)\hat{\theta}_{1i}}{a_T} + J_{mi}\dot{\omega}_{di} \\ &+ \frac{T_{ni}(\omega_{di})}{a_T} + k_{fi}\omega_{di}. \end{aligned} \tag{30}$$

makes the origin of (29) UGES when $\hat{\theta}_{1i} = \theta_{1i}$. The rudder model is linearly time-variant and the error dynamic is given by:

$$m_i\dot{\tilde{\delta}} = a_i(t)\left(\tilde{\delta} + \delta_{di}\right) + b_i u_{cmd\delta i} - m_i\dot{\delta}_{di} \tag{31}$$

where $\tilde{\delta} := \delta_i - \delta_{di}$, a_i, b_i are a known scalar parameter bounded away from zero, and the controller

$$b_i u_{cmd\delta i} := -K_\delta \tilde{\delta} - a_i(t)\left(\tilde{\delta} + \delta_{di}\right) + m_i \dot{\delta}_{di} \qquad (32)$$

makes the origin of (31) UGES. The parameters for the actuator model and controllers are: $a_T = 1$, $J_{mi} = 10^{-2}$, $k_{fi} = 10^{-4}$, $a_i = -10^{-4}$, $b_i = 10^{-5}$, $m_i = 10^{-2}$, $K_{\omega p} = 5 \cdot 10^{-3}$ and $K_\delta = 10^{-3}$

A virtual controller τ_c that stabilizes the system (27) uniformly, globally and exponentially, for some physically limited yaw rate, is proposed in (Lindegaard and Fossen, 2003) and given by

$$\tau_c := -K_i R^T(\psi_p)\xi - K_p R^T(\psi_p)\eta_e - K_d \nu, \qquad (33)$$

where (27) is augmented with the integral action described by, $\dot{\xi} = \eta_e$. Thus Assumption 2 concerning high- and low- level control is satisfied. The cost function designed for the optimization problem, (5), is:

$$J(u) := \sum_{i=1}^{3} k_i \left|\omega_i\right| \omega_i^2 + k_{i2}\omega_i^2 + \sum_{i=1}^{2} q_i \delta_i^2 - \varsigma \sum_{i=1}^{3} \lg(-\omega_i + 18)$$
$$-\varsigma \sum_{i=1}^{3} \lg(\omega_i + 18) - \varsigma \sum_{i=1}^{2} \lg(-\delta_i + 35) - \varsigma \sum_{i=1}^{2} \lg(\delta_i + 35),$$
$$\varsigma = 0.05, \quad k_1 = k_2 = 0.01, \quad k_3 = 0.02, \quad k_{i2} = 10^{-3},$$
$$q_1 = q_2 = 2500.$$

By investigating the given specifications of the system we can see that the Assumption 3 is also satisfied locally, since u is bounded. The gain matrices are chosen as follows: $K_p := M \cdot diag(3.13, 3.13, 12.5)10^{-2}$, $K_d := M \cdot diag(3.75, 3.75, 7.5)10^{-1}$, $K_I := M \cdot diag(0.2, 0.2, 4)10^{-}$ $A_{\hat{x}} := 10$, $\Gamma_{\tilde{x}} := I_{9\times 9}$, $\Gamma_{\theta_2}^{-1} := 10^{-4} diag\left(1, 1, 10\right)$, $A_{\hat{u}} := 2I_{5\times 5}$, $\Gamma_{\theta_1} := 10^{-3}$, $\Gamma_\eta := diag(10^3, 10^3, 3)$ and $\Gamma := \left(\mathbb{H}_{\hat{\theta}}^{\mathrm{T}} W \mathbb{H}_{\hat{\theta}} + \varepsilon I\right)^{-1}$ where $W := diag\left(1, 1, 1, 1, 1, 0.9, 0.9, 0.7\right)$ and $\varepsilon := 10^{-9}$.

The thruster loss vector θ_1 and $\hat{\theta}_1$ are given in Figure 6, $\theta_2 := (0.05, 0.08, 0.02)$ and $\hat{\theta}_2$ are given in Figure 7.

The simulation results are presented in the Figures 2-8. The control objective is satisfied and the commanded virtual controls are tracked by the forces generated by the adaptive control allocation law: see Figure 5. Note that there are some deviations since ω saturates from $0 - 230s$ and since the loss parameter has changed at ca. $420s$. Also note that the parameter estimates $\hat{\theta}_1$ only converge to the true values when the ship is moving and the thrust loss is not zero. The simulations are carried out in a discrete MATLAB environment with a sampling rate of $20Hz$

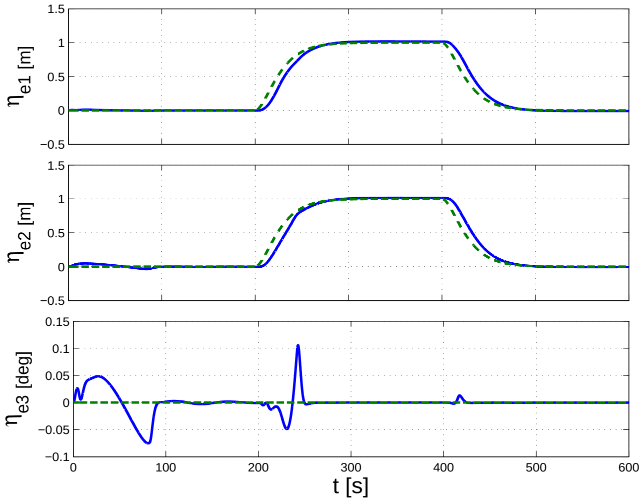

Figure 2: Desired (dashed) and actual ship positions (solid).

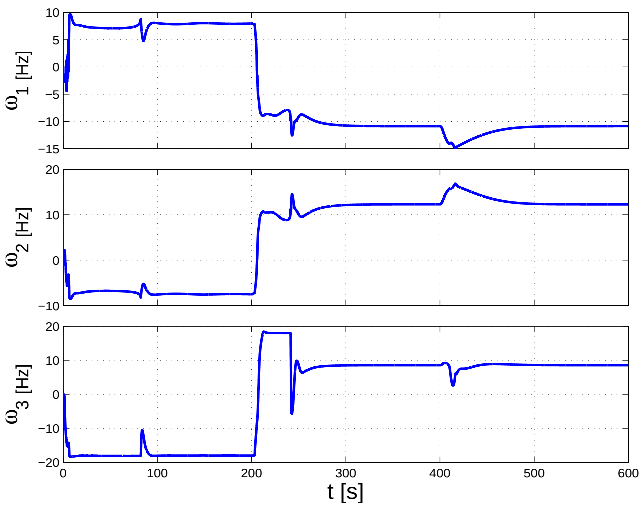

Figure 3: Actual propeller velocities

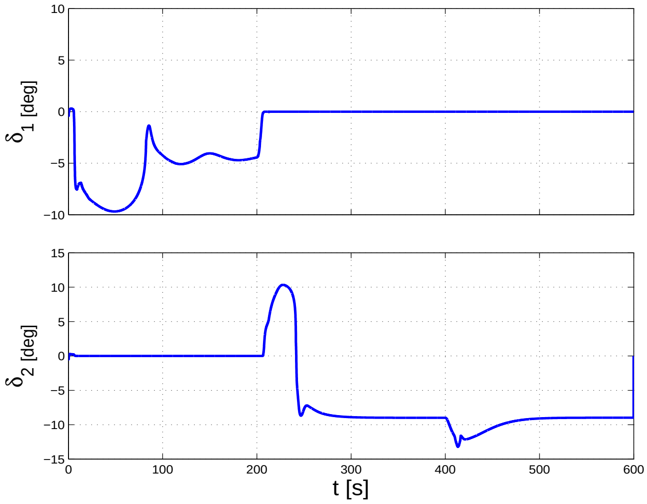

Figure 4: Actual rudder deflection

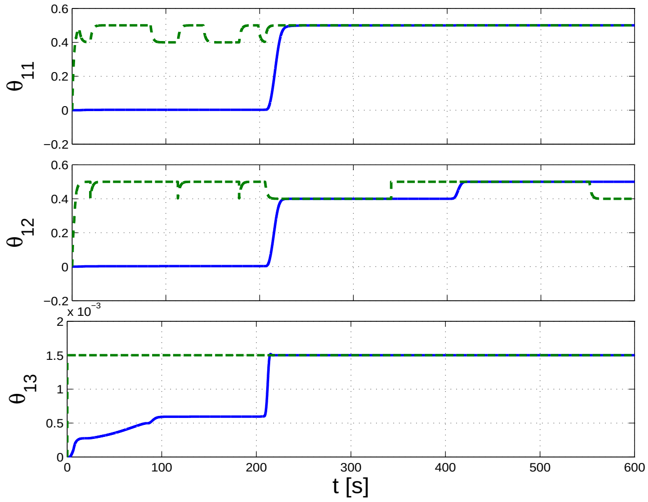

Figure 6: Actual (dashed) and estimated (solid) loss parameters

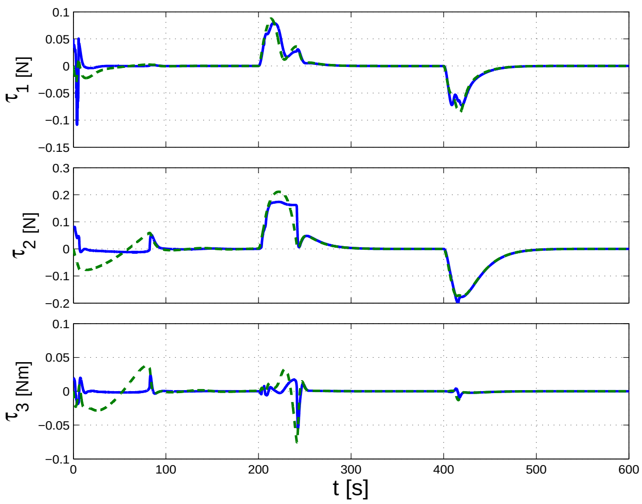

Figure 5: The virtual control (dashed) and actual (solid) forces generated by the actuators

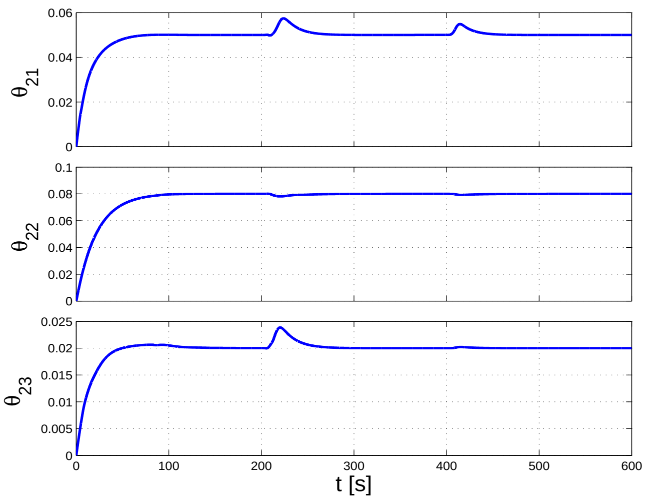

Figure 7: Effector model parameter estimates

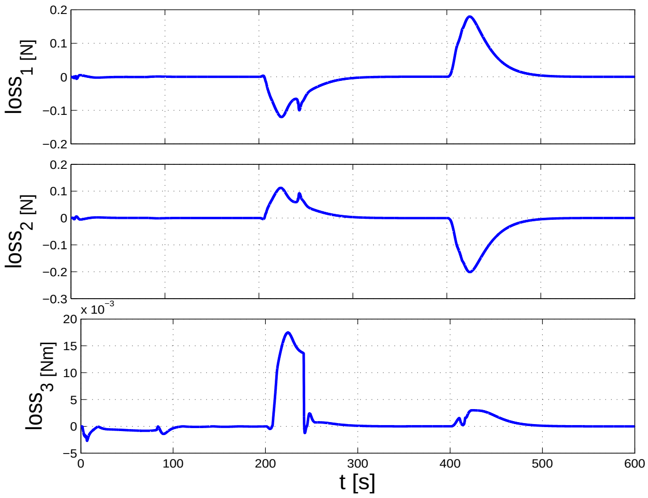

Figure 8: Actual thrust loss

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